Kansas State University | Salina

Statistics Workbook

Certified Inspector Training Program

Kansas Department of Transportation

Statistics Workbook

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Statistics Handbook

Certified Inspector Training Program

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INTRODUCTION

Rick Barezinsky, P.E.

Assistant Bureau Chief, Construction and Materials

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Course Format

» Topics

– Basic Statistics (Section 5.2.1)
– Quality Level Analysis (Section 5.2.1)
– Statistical Comparison of Quality Control and Verification Tests (Section 5.2.6)

» Test

– Open Book; Open Note
– Multiple Choice Test
– 70% Required to Pass

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Need Help? Ask Questions Any Time

» Discussion Board

– Leave questions on the discussion board
– View other questions and responses

» Response Time

– Within 48 hours (hopefully much sooner)

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BASIC STATISTICS

KDOT Construction Manual

Section 5.2.1

Pages 3-6

5

Basic Statistics
Objectives

- Calculate basic statistical measures
- Perform basic statistical calculations on calculator
- Name methods to generate random numbers

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Basic Statistics
Definition of Statistics

- **Statistics**
  - » science of interpreting numerical data that has been collected systematically, summarized, and tabulated

- By knowing what has happened or the way things are helps in
  - making decisions;
  - making predictions; and
  - Taking steps to improve processes

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Basic Statistics
Measures and Procedures

Central value measures

- *Average or Mean (5.1)*
- *4-Point Moving Average (5.2)*

Variability measures

- *Range (5.3)*
- *Standard Deviation (5.4)*
- *Sample Variance (5.6)*

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Basic Statistics
Common Terms

- **Data Set** - a group of data (numbers)

  » Numbers usually represented as variable (x_i)

  – x_1 - represents first number in data set,

  – x_2 - represents second number in data set, etc…

  » Total number of variables represented as n

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Basic Statistics
Random Sampling

- **Random number (5.2.2.2)**

  » A number selected entirely by chance as from a table of random numbers.

  – Note: Other methods of generating random numbers, such as with scientific calculators and spreadsheets must be approved by the District Construction/Materials Engineer.

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Basic Statistics
Random Sampling

- **Sample (5.2.2.2)**
  - » A small part of a lot or a sublot which represents the whole.
- **Random Sample**
  - » Use the **random number** to determine where to take your **sample**.
- Random sampling helps eliminate bias and ensures reliability of our data.

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Basic Statistics
Measures and Procedures

Central value measures

- *Average or Mean (5.1)*
- *4-Point Moving Average (5.2)*

Variability measures

- *Range (5.3)*
- *Standard Deviation (5.4)*
- *Sample Variance (5.6)*

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Basic Statistics
5.1. Average or Mean ($\bar{x}$)
Seat Location

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

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Basic Statistics
5.1. Average or Mean ($\bar{x}$)

Data Point	Random Location Seat Location (1-30)	Height (in)
X_1	29	
X_2	24	
X_3	20	
X_4	11	
X_5	21	
X_6	12	

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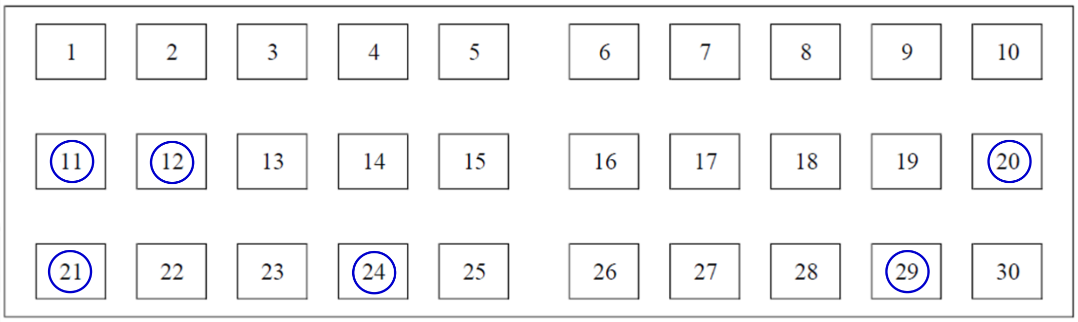
Basic Statistics
5.1. Average or Mean ($\bar{x}$)
Seat Location



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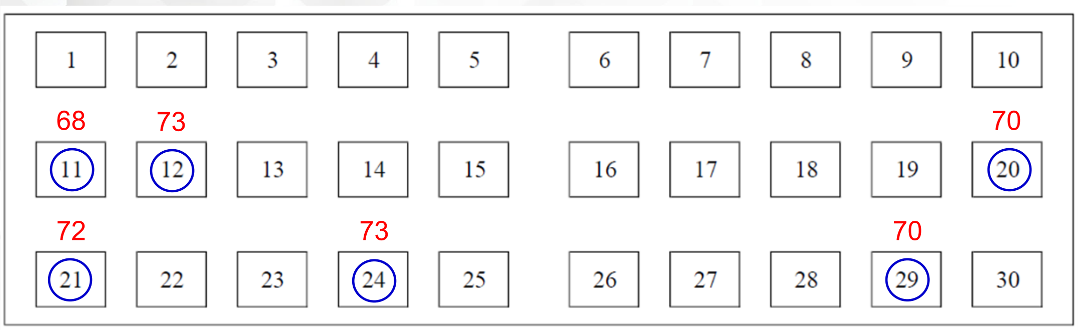
Basic Statistics
5.1. Average or Mean ($\bar{x}$)
Seat Location



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Basic Statistics
5.1. Average or Mean ($\bar{x}$)

Data Point	Random Location Seat Location (1-30)	Height (in)
X_1	29	70
X_2	24	73
X_3	20	70
X_4	11	68
X_5	21	72
X_6	12	73

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Basic Statistics
5.1. Average or Mean ($\bar{x}$)

Data Point	Height (in)
X_1	70
X_2	73
X_3	70
X_4	68
X_5	72
X_6	73

$n = 6$

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Basic Statistics
5.1. Average or Mean ($\bar{x}$)

- The total sum of all variables (x_i) divided by the number of variables (n)

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\Sigma x_i}{n}$$

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Basic Statistics
5.1. Average or Mean ($\bar{x}$)

Data Point	Height (in)
X_1	70
X_2	73
X_3	70
X_4	68
X_5	72
X_6	73
n = 6	426

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{n}$$

$$\bar{x} = \frac{70 + 73 + 70 + 68 + 72 + 73}{6}$$

$$\bar{x} = \frac{426}{6} = 71 \text{ in.}$$

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Basic Statistics
Measures and Procedures

Central value measures

- *Average or Mean (5.1)*
- *4-Point Moving Average (5.2)*

Variability measures

- *Range (5.3)*
- *Standard Deviation (5.4)*
- *Sample Variance (5.6)*

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Basic Statistics
5.2. Moving Average (x_{ma})

- Average computed based on a fixed set of continuous data points

- The 4-point moving average will be:

$$x_{ma_i} = \frac{xi + x(i-1) + x(i-2) + x(i-3)}{4}$$

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Basic Statistics
5.2. Moving Average (x_{ma})

Data Point	Height (in)
X_1	70
X_2	73
X_3	70
X_4	68
X_5	72
X_6	73

n = 6

$$x_{ma_i} = \frac{xi + x(i-1) + x(i-2) + x(i-3)}{4}$$

$$x_{ma_4} = \frac{x_4 + x_3 + x_2 + x_1}{4}$$

$$x_{ma_4} = \frac{68 + 70 + 73 + 70}{4}$$

$$x_{ma_4} = \frac{281}{4} = 70.25 \text{ in.}$$

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Basic Statistics
5.2. Moving Average (x_{ma})

Data Point	Height (in)
X_1	70
X_2	73
X_3	70
X_4	68
X_5	72
X_6	73

n = 6

$$x_{ma_i} = \frac{xi + x(i-1) + x(i-2) + x(i-3)}{4}$$

$$x_{ma_5} = \frac{x_5 + x_4 + x_3 + x_2}{4}$$

$$x_{ma_5} = \frac{72 + 68 + 70 + 73}{4}$$

$$x_{ma_5} = \frac{283}{4} = 70.75 \text{ in.}$$

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Basic Statistics

5.2. Moving Average (x_{ma})

Data Point	Height (in)
X_1	70
X_2	73
X_3	70
X_4	68
X_5	72
X_6	73

n = 6

$$x_{ma_i} = \frac{xi + x(i-1) + x(i-2) + x(i-3)}{4}$$

$$x_{ma_6} = \frac{x_6 + x_5 + x_4 + x_3}{4}$$

$$x_{ma_6} = \frac{73 + 72 + 68 + 70}{4}$$

$$x_{ma_6} = \frac{283}{4} = 70.75 \text{ in.}$$

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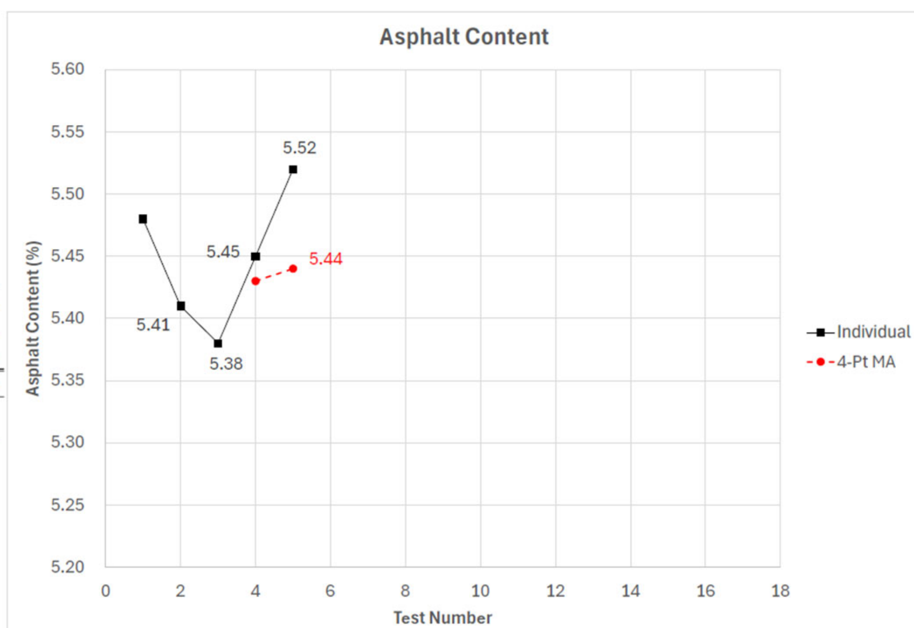
2	5.41
3	5.38
4	5.45
5	5.52
Sum	21.76
n	4
4pt MA_5	5.44



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Basic Statistics
Measures and Procedures

Central value measures
- *Average or Mean (5.1)*
- *4-Point Moving Average (5.2)*

Variability measures
- *Range (5.3)*
- *Standard Deviation (5.4)*
- *Sample Variance (5.6)*

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Basic Statistics
5.3. Range (R)

- The difference between the largest and the smallest values in a data set.
- A simple measure of variability.

$$R = x_{\max} - x_{\min}$$

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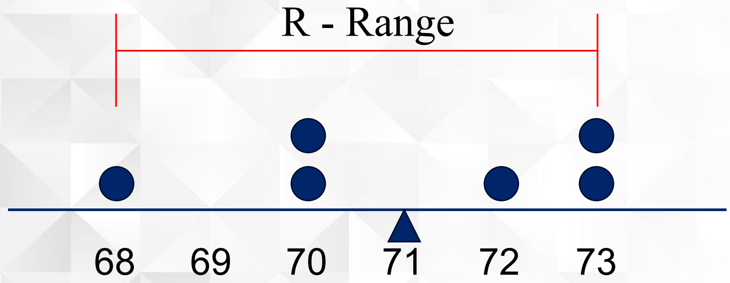
Basic Statistics
5.3. Range (R)



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Basic Statistics
5.3. Range (R)

Data Point	Height (in)	Max/Min
X_1	70	
X_2	73	Max
X_3	70	
X_4	68	Min
X_5	72	
X_6	73	Max

n = 6

$$R = x_{\text{max}} - x_{\text{min}}$$

$$R = 73 - 68$$

$$R = 5 \text{ in.}$$

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Basic Statistics
Measures and Procedures

Central value measures
- *Average or Mean (5.1)*
- *4-Point Moving Average (5.2)*

Variability measures
- *Range (5.3)*
- *Standard Deviation (5.4)*
- *Sample Variance (5.6)*

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Basic Statistics
5.4. Standard Deviation (S)

- Measure of the variation about the average of the data set
- Provides a better measure of variability than range
- 2 types of Standard Deviation depending on sample size
  - » **Sample Standard Deviation (30 samples or less)**
  - » Population Standard Deviation (more than 30 samples)

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Basic Statistics
5.4. Sample Standard Deviation (s)

- The root mean square of the deviation from the mean.

- Typically used when the sample size is 30 or less.

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

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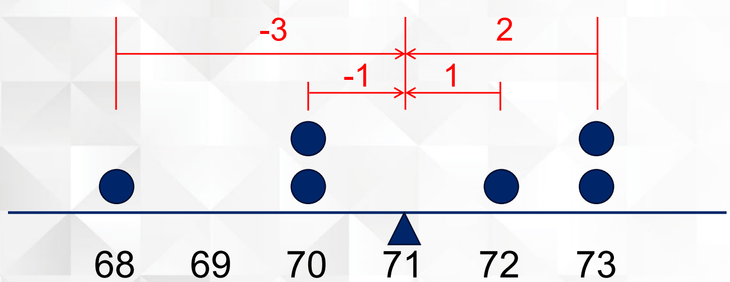
Basic Statistics
5.4. Standard Deviation (S)



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Basic Statistics

5.4. Standard Deviation (s)

Data Point	Height x_i	Mean $\bar{x}$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
X_1	70	71	-1	1
X_2	73	71	2	4
X_3	70	71	-1	1
X_4	68	71	-3	9
X_5	72	71	1	1
X_6	73	71	2	4
n = 6			$\sum$ =	20

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{20}{6-1}}$$

$$s = \sqrt{\frac{20}{5}} = \sqrt{4} = 2 \text{ in.}$$

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Basic Statistics

Measures and Procedures

Central value measures

- *Average or Mean (5.1)*
- *4-Point Moving Average (5.2)*

Variability measures

- *Range (5.3)*
- *Standard Deviation (5.4)*
- *Sample Variance (5.6)*

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Basic Statistics
5.6. Variance (s^2)

- The square of the sample standard deviation

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

$s = 2$ in.

$s^2 = 2^2 = 4$ in.^{2}

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Part V
Section 5.2.1. Examples

5. INTRODUCTORY STATISTICAL TERMS

5.1. Average or Mean ($\bar{x}$): Arithmetic mean or average determined for a number of variables (x_i) as below:

$$\bar{x} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{\sum X_i}{n} \qquad (1)$$

5.1.1. *Example: Find the arithmetic mean or average for the asphalt content of six Superpave mix sublots given as: 5.4, 5.8, 6.2, 5.4, 5.4 and 6.0%.*

$$\bar{x} = \frac{5.4 + 5.8 + 6.2 + 5.4 + 5.4 + 6.0}{6} = \frac{34.2}{6} = 5.7$$

5.1.2. *Example: Find the arithmetic mean or average for the percent air in the concrete mix of six sublots given as: 6.6, 6.2, 5.5, 7.8, 6.9 and 6.6%.*

$$\bar{x} = \frac{6.6 + 6.2 + 5.5 + 7.8 + 6.9 + 6.6}{6} = \frac{39.6}{6} = 6.6$$

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Part V
Section 5.2.1. Examples

5.2.1. *Example: Find the 4-point moving average for the above asphalt content data:*

Asphalt content (%)	4-point moving average	
5.4	-	$5.4 + 5.8 + 6.2 + 5.4 = 22.8 \div 4 = 5.7$
5.8	-	$5.8 + 6.2 + 5.4 + 5.4 = 22.8 \div 4 = 5.7$
6.2	-	$6.2 + 5.4 + 5.4 + 6.0 = 23.0 \div 4 = 5.8$
5.4		
5.4		
6.0		

5.2.2. *Example: Find the 4-point moving average for the above percent air content data:*

Air content (%)	4-point moving average	
6.6	-	$6.6 + 6.2 + 5.5 + 7.8 = 26.1 \div 4 = 6.5$
6.2	-	$6.2 + 5.5 + 7.8 + 6.9 = 26.4 \div 4 = 6.6$
5.5	-	$5.5 + 7.8 + 6.9 + 6.6 = 26.8 \div 4 = 6.7$
7.8		
6.9		
6.6		

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Part V
Section 5.2.1. Examples

5.4. Sample Standard Deviation (s): Standard deviation is the root mean square of the deviation from the mean. This is a better measure of variability than range and is computed as below:

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} \qquad (4)$$

where, n is the sample size.

5.4.1. *Example: Find the standard deviation for the asphalt content data given below:*

x_i	$\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
5.4	5.7	-0.3	0.09
5.8	5.7	0.1	0.01
6.2	5.7	0.5	0.25
5.4	5.7	-0.3	0.09
5.4	5.7	-0.3	0.09
6.0	5.7	0.3	0.09
n = 6			$\Sigma = 0.62$

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{0.62}{6-1}} = 0.35$$

X_1: 5.4 - 5.7 = -0.3^2 = 0.09
X_2: 5.8 - 5.7 = 0.1^2 = 0.01
X_3: 6.2 - 5.7 = 0.5^2 = 0.25
X_4: 5.4 - 5.7 = -0.3^2 = 0.09
X_5: 5.4 - 5.7 = -0.3^2 = 0.09
X_6: 6.0 - 5.7 = 0.3^2 = 0.09

0.62

$$s = \sqrt{\frac{0.62}{6 - 1}} \qquad s = \sqrt{\frac{0.62}{5}}$$

$$s = \sqrt{0.124} = 0.35$$

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Part V
Section 5.2.1. Examples

5.4.2. *Example: Find the standard deviation for the air content data given below:*

x_i	$\bar{x}$	$x_i-\bar{x}$	$(x_i-\bar{x})^2$
6.6	6.6	0.0	0.00
6.2	6.6	-0.4	0.16
5.5	6.6	-1.1	1.21
7.8	6.6	1.2	1.44
6.9	6.6	0.3	0.09
6.6	6.6	0.0	0.00
n =6			Σ = 2.90

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{2.90}{6-1}} = 0.76$$

Page 5/26 5.2.1 Revised 2022

X_1: 6.6 – 6.6 = 0.0^2 =0.00
X_2: 6.2 – 6.6 = -0.4^2 =0.16
X_3: 5.5 – 6.6 = -1.1^2 =1.21
X_4: 7.8 – 6.6 = 1.2^2 =1.44
X_5: 6.9 – 6.6 = 0.3^2 =0.09
X_6: 6.6 – 6.6 = 0.0^2 =0.00
2.90

$$s = \sqrt{\frac{2.90}{6-1}} \quad s = \sqrt{\frac{2.90}{5}}$$

$$s = \sqrt{0.58} = 0.76$$

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Basic Statistics
Calculator Usage

- Refer to Owners Manual
- Use YouTube
- Search for calculator model + "statistics"
- Examples:
  - TI-30XA - press [2nd] then [DATA]
  - TI-84 – press [MODE] then scroll to Stat Diagnostics and select "On"
  - Canon LS-82Z – Press [SHIFT] then [RCL]
  - Casio fx-260 - Press [MODE] then [.]
  - Sharp EL-W516X – Press [MODE] then [1] then [0]

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Basic Statistics
Calculator Usage

- Entering Data Points
  - Enter the number and press [DATA]
  - Continue for your entire data set.
- Number of entries in your Dataset
  - n
- Mean or Average
  - $\bar{x}$
- Sample Standard Deviation
  - s or $\sigma_{n\text{-}1}$

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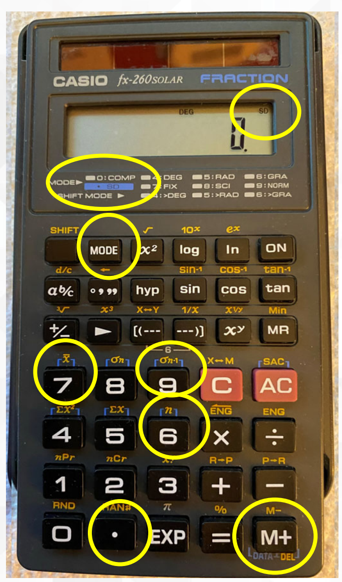
Basic Statistics
Calculator Usage

- Casio fx-260



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Basic Statistics
Objectives

- Calculate basic statistical measures
- Perform basic statistical calculations on calculator
- Name methods to generate random numbers

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NORMAL DISTRIBUTION CURVE

KDOT Construction Manual

Section 5.2.1

Pages 7-10

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1

Normal Distribution Curve
Objectives

- List at least 4 characteristics of normal distribution curves
- Describe how normal distribution curves relate to process control
- Describe how normal distribution curves serve as a basis for statistical control charts

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Normal Distribution Curve
What is a Distribution?

- A listing or function showing all the possible values of the data set and how often they occur
- When organized, they're often ordered from smallest to largest, broken into reasonable sized groups and then put into graphs or charts to examine the shape, center and amount of variability in the data

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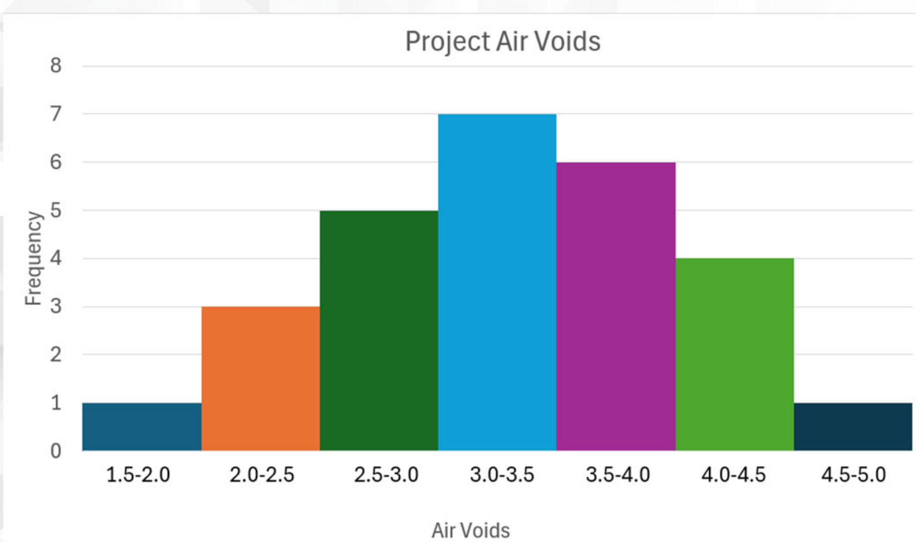
Normal Distribution Curve
What is a Distribution?



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Normal Distribution Curve
Characteristics

- A typical "bell-shaped" symmetrical curve which usually describes the distribution of engineering measurements
- The **average or mean** is the curve center
- The **standard deviation** determines the curve shape

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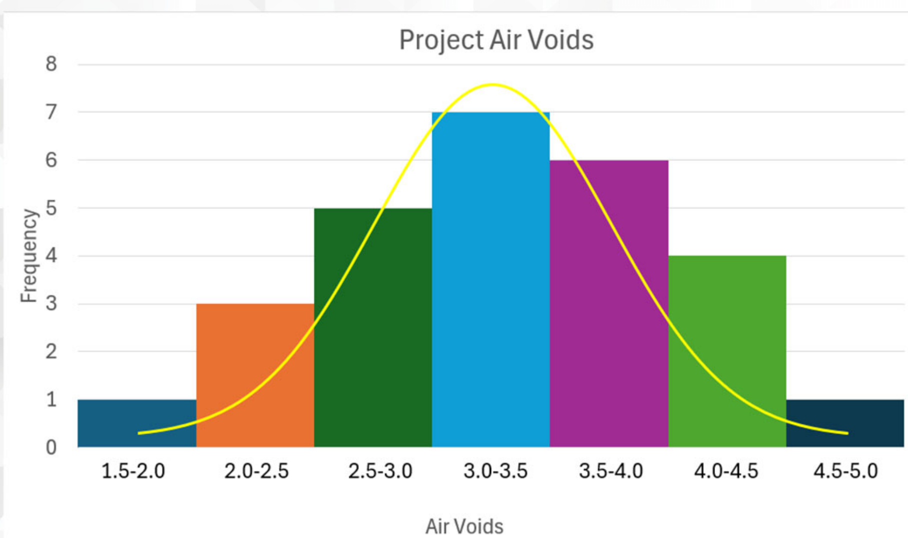
Normal Distribution Curve
What is a Distribution?



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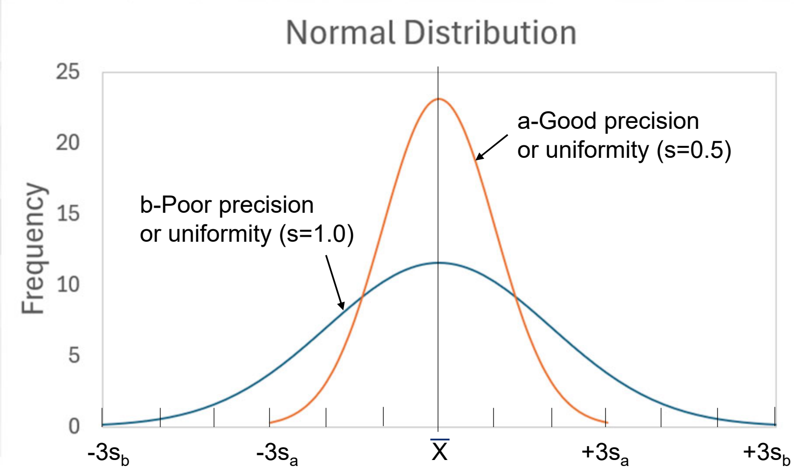
Normal Distribution Curve
Characteristics



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Normal Distribution Curve
Characteristics

- **68.26%** of data falls within **± 1** standard deviation of the average
- **95.44%** of data falls within **± 2** standard deviations of the average
- **99.74%** of data falls within **± 3** standard deviations of the average

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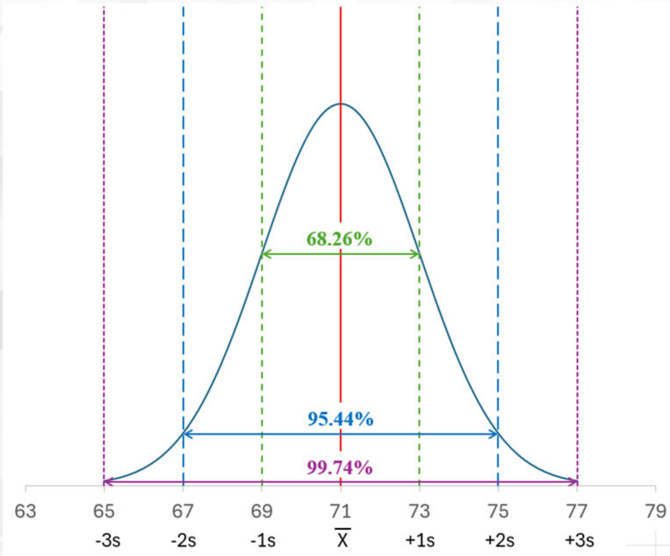
Normal Distribution Curve
Characteristics



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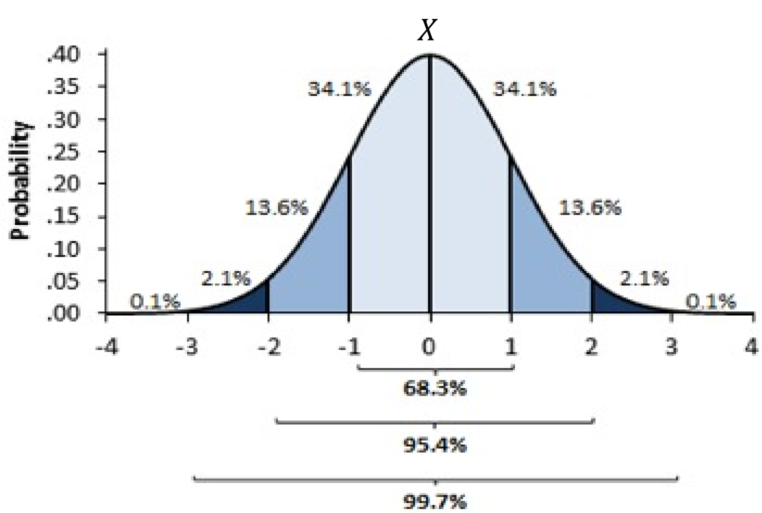
g. Compressive Strength



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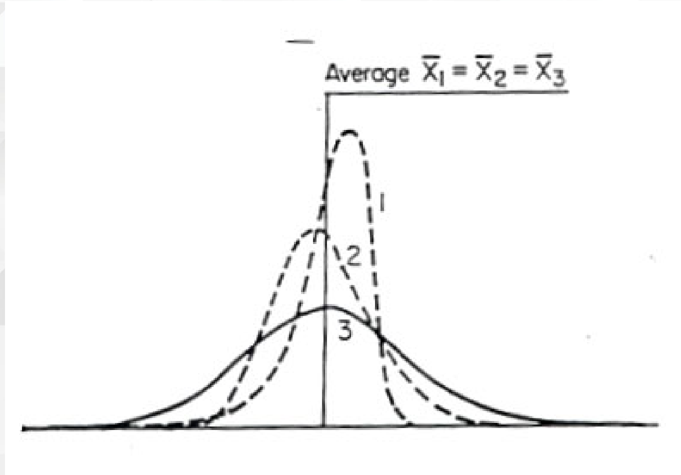
Normal Distribution Curve
Different Distributions can Share the Same Mean



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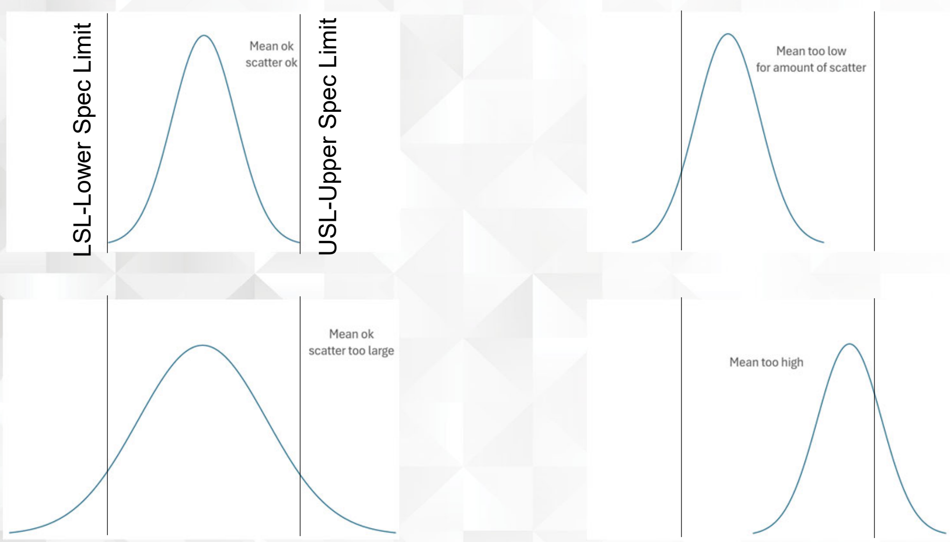
Normal Distribution Curve
Used for Process Control



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Normal Distribution Curve
Used for Statistical Control Charts

- Normal distribution curves can serve as the basis for statistical control charts
- The upper specification limit (USL) and lower specification limit (LSL) can be based on multiples of standard deviation

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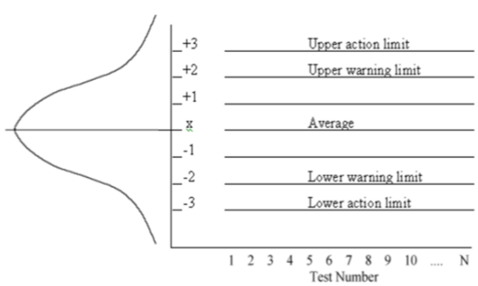
Normal Distribution Curve
Used for Statistical Control Charts



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Normal Distribution Curve
Objectives

- List at least 4 characteristics of normal distribution curves
- Describe how normal distribution curves relate to process control
- Describe how normal distribution curves serve as a basis for statistical control charts

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QUALITY LEVEL ANALYSIS

KDOT Construction Manual

Section 5.2.1

Pages 12-26

1

Quality Level Analysis
Objectives

- Define Quality Level Analysis
- State and calculate upper and lower quality index formulas
- Use a table to find Percent Within Limits (PWL)
- Select and apply proper analyses

2

Quality Level Analysis
Definitions

- A statistical procedure that provides a method of estimating the percentage of each lot or sublot of material, product item of construction, or completed construction that may be expected to be within *specified tolerance limits.*

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Quality Level Analysis
Definitions

- **Percent Within Limits** (**PWL**) – A quality measure that estimates the percentage of material or product that is within specified tolerance limits for a given quality characteristic using the sample mean and the sample standard deviation.

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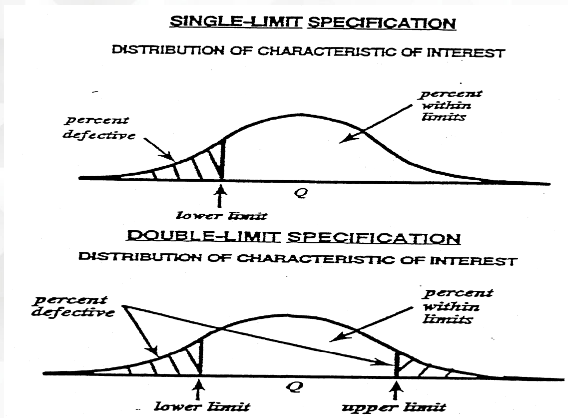
Quality Level Analysis
Concept of Percent Defective

Page 13 (5.2.1)



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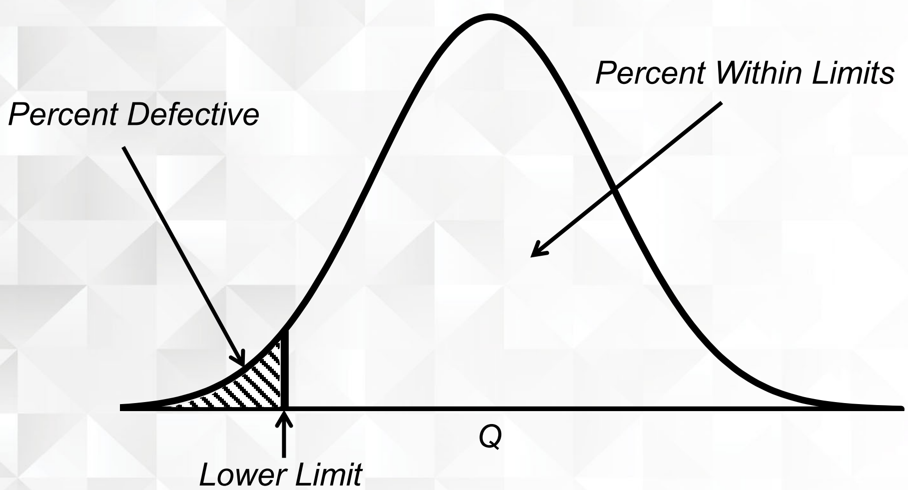
Quality Level Analysis
Concept of Percent Defective

Single Limit Specification



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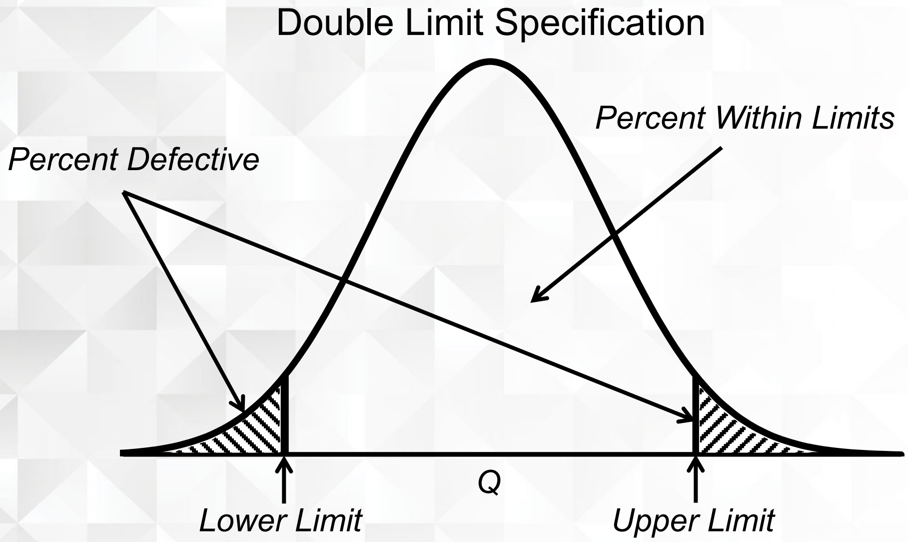
Quality Level Analysis
Concept of Percent Defective



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Quality Level Analysis
Upper Quality Index

- **Upper Quality Index** (Q_u) - subtract the average from the upper specification limit and divide by the sample standard deviation

$$Q_u = \frac{(USL - \bar{X})}{s}$$

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Quality Level Analysis
Lower Quality Index

- **Lower Quality Index** (Q_L) - subtract the lower specification limit from the average and divide by the sample standard deviation
  - » Same as Quality Index (Q) for single limit specifications with lower-limit

$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

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Quality Level Analysis
Percent Within Limits

- **Percent Within Limits** (**PWL**) is determined from Table 2 (Pages 18-26) after the Quality Index(es) have been computed
  - » If Quality Index is greater than 3.76, then PWL=100%
  - » If Quality Index is a negative number, the Percent Within Limits is equal to:
    100% - (PWL from Table 2)

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Quality Level Analysis
Percent Within Limits

$Q = 1.00$

$n = 5$

$PWL = 83.64\%$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.90	78.45	80.00	80.62	80.93	81.10	81.21	81.28	81.33	81.46	81.50	81.54	81.57	81.58
0.91	78.89	80.33	80.93	81.22	81.39	81.49	81.56	81.61	81.73	81.77	81.81	81.83	81.85
0.92	79.34	80.67	81.23	81.51	81.67	81.77	81.84	81.89	82.00	82.04	82.08	82.10	82.11
0.93	79.81	81.00	81.54	81.81	81.96	82.05	82.12	82.16	82.27	82.31	82.34	82.36	82.37
0.94	80.27	81.33	81.84	82.10	82.24	82.33	82.39	82.44	82.54	82.57	82.60	82.62	82.63
0.95	80.75	81.67	82.14	82.39	82.52	82.61	82.67	82.71	82.80	82.84	82.86	82.88	82.89
0.96	81.25	82.00	82.45	82.67	82.80	82.88	82.94	82.97	83.06	83.10	83.12	83.13	83.14
0.97	81.75	82.33	82.75	82.96	83.08	83.15	83.21	83.24	83.32	83.35	83.37	83.39	83.39
0.98	82.26	82.67	83.04	83.24	83.35	83.43	83.47	83.51	83.58	83.61	83.63	83.64	83.64
0.99	82.79	83.00	83.34	83.52	83.63	83.69	83.74	83.77	83.84	83.86	83.88	83.88	83.89
1.00	83.33	83.33	83.64	83.80	83.90	83.96	84.00	84.03	84.09	84.11	84.12	84.13	84.13
1.01	83.89	83.67	83.93	84.08	84.17	84.22	84.26	84.28	84.34	84.36	84.37	84.37	84.38
1.02	84.47	84.00	84.22	84.36	84.44	84.49	84.52	84.54	84.59	84.60	84.61	84.62	84.62
1.03	85.07	84.33	84.52	84.63	84.70	84.75	84.77	84.79	84.83	84.85	84.85	84.85	84.85

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Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.35	100.00	95.00	92.98	92.37	92.08	91.90	91.78	91.70	91.48	91.39	91.31	91.24	91.19
1.36	100.00	95.33	93.21	92.58	92.27	92.09	91.96	91.88	91.65	91.56	91.47	91.40	91.35
1.37	100.00	95.67	93.44	92.78	92.46	92.27	92.14	92.05	91.82	91.72	91.63	91.56	91.51
1.38	100.00	96.00	93.67	92.98	92.65	92.45	92.32	92.23	91.99	91.88	91.79	91.72	91.67
1.39	100.00	96.33	93.90	93.18	92.83	92.63	92.49	92.40	92.15	92.04	91.95	91.88	91.82
1.40	100.00	96.67	94.12	93.37	93.02	92.81	92.67	92.56	92.31	92.20	92.10	92.03	91.98
1.41	100.00	97.00	94.34	93.57	93.20	92.98	92.83	92.73	92.47	92.36	92.26	92.18	92.13
1.42	100.00	97.33	94.56	93.76	93.38	93.15	93.00	92.90	92.63	92.51	92.41	92.33	92.27
1.43	100.00	97.67	94.77	93.95	93.55	93.32	93.17	93.06	92.78	92.66	92.56	92.48	92.42
1.44	100.00	98.00	94.98	94.13	93.73	93.49	93.33	93.22	92.93	92.81	92.70	92.62	92.56
1.45	100.00	98.33	95.19	94.32	93.90	93.65	93.49	93.37	93.08	92.96	92.85	92.76	92.70
1.46	100.00	98.67	95.40	94.50	94.07	93.81	93.65	93.53	93.23	93.10	92.99	92.90	92.84
1.47	100.00	99.00	95.61	94.67	94.23	93.97	93.80	93.68	93.37	93.25	93.13	93.04	92.98
1.48	100.00	99.33	95.81	94.85	94.40	94.13	93.96	93.83	93.52	93.39	93.27	93.18	93.12
1.49	100.00	99.67	96.01	95.02	94.56	94.29	94.11	93.98	93.66	93.52	93.40	93.31	93.25
1.50	100.00	100.00	96.20	95.19	94.72	94.44	94.26	94.13	93.80	93.66	93.54	93.45	93.38
1.51	100.00	100.00	96.39	95.36	94.87	94.59	94.40	94.27	93.94	93.80	93.67	93.58	93.51
1.52	100.00	100.00	96.58	95.53	95.03	94.74	94.55	94.41	94.07	93.93	93.80	93.71	93.64
1.53	100.00	100.00	96.77	95.69	95.18	94.88	94.69	94.55	94.20	94.06	93.93	93.83	93.76
1.54	100.00	100.00	96.95	95.85	95.33	95.03	94.83	94.69	94.33	94.19	94.05	93.96	93.89
1.55	100.00	100.00	97.13	96.00	95.48	95.17	94.97	94.82	94.46	94.31	94.18	94.08	94.01
1.56	100.00	100.00	97.31	96.16	95.62	95.31	95.10	94.95	94.59	94.44	94.30	94.20	94.13
1.57	100.00	100.00	97.48	96.31	95.76	95.44	95.23	95.08	94.71	94.56	94.42	94.32	94.25

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Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
3.60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.61	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.62	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.63	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.64	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.65	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.66	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.67	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.68	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.69	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.71	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.72	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.73	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.74	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.76	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

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5.2.1

The estimates of lot percent within limits (PWL) provided in the tables are obtained by numerically integrating the beta distribution function corresponding to Quality Index (Q) and Sample Size (N).

To find PWL from the tables, compute Q from the sample mean and sample standard deviation with unknown population variability, and the lower or upper specification limits.

To find the PWL for a negative Quality Index, first get the PWL for the positive value of the Quality Index from the tables and subtract the result from 100.

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Quality Level Analysis
Percent Within Limits

$Q_L = -0.55$

$n = 4$

Enter Table 2 with:

$Q_L = 0.55$

$PWL_L = 68.33$

$PWL_L = 100 - 68.33$

$PWL_L = 31.67$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.45	62.74	65.00	65.84	66.27	66.51	66.67	66.79	66.87	67.08	67.16	67.24	67.29	67.33
0.46	63.04	65.33	66.19	66.62	66.87	67.03	67.14	67.22	67.43	67.52	67.60	67.65	67.69
0.47	63.34	65.67	66.53	66.96	67.22	67.38	67.49	67.58	67.79	67.88	67.96	68.01	68.05
0.48	63.65	66.00	66.88	67.31	67.57	67.73	67.85	67.93	68.15	68.23	68.31	68.37	68.40
0.49	63.95	66.33	67.22	67.66	67.92	68.08	68.20	68.28	68.50	68.59	68.67	68.72	68.76
0.50	64.25	66.67	67.56	68.00	68.26	68.43	68.55	68.63	68.85	68.94	69.02	69.07	69.11
0.51	64.56	67.00	67.90	68.35	68.61	68.78	68.90	68.98	69.20	69.29	69.37	69.43	69.46
0.52	64.87	67.33	68.24	68.69	68.96	69.13	69.24	69.33	69.55	69.64	69.72	69.77	69.81
0.53	65.18	67.67	68.58	69.04	69.30	69.47	69.59	69.68	69.90	69.99	70.07	70.12	70.16
0.54	65.49	68.00	68.92	69.38	69.64	69.82	69.93	70.02	70.24	70.33	70.41	70.47	70.51
0.55	65.80	68.33	69.26	69.72	69.99	70.16	70.28	70.36	70.59	70.68	70.76	70.81	70.85
0.56	66.12	68.67	69.60	70.06	70.33	70.50	70.62	70.71	70.93	71.02	71.10	71.15	71.19
0.57	66.43	69.00	69.94	70.40	70.67	70.84	70.96	71.05	71.27	71.36	71.44	71.49	71.53
0.58	66.75	69.33	70.27	70.74	71.01	71.18	71.30	71.39	71.61	71.70	71.78	71.83	71.87
0.59	67.07	69.67	70.61	71.07	71.34	71.52	71.64	71.72	71.95	72.04	72.11	72.17	72.21

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Quality Level Analysis
Two Types of Analyses

- There are two types: double-limit and single-limit specifications
- Steps in analysis for a double-limit specification on Page 13
- Steps in analysis for a single-limit specification with a lower-limit specification shown on Page 14

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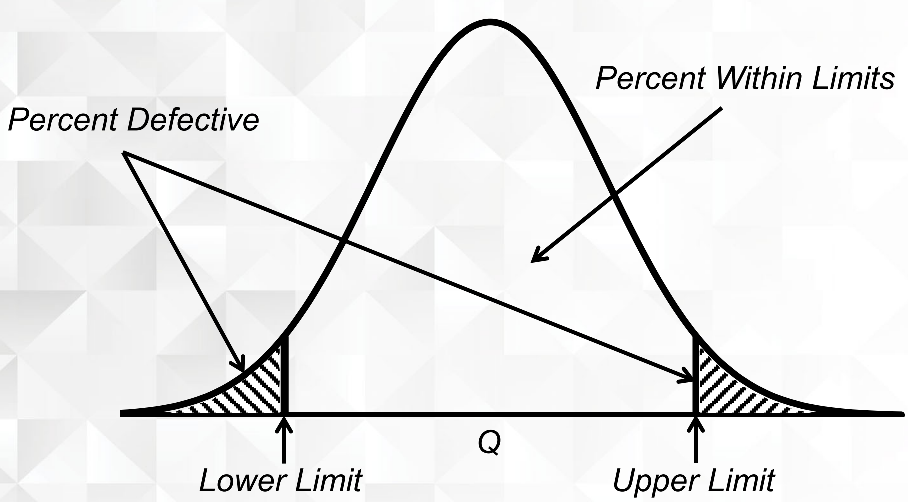
Quality Level Analysis
Double Limit Specification



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Quality Level Analysis
Double Limit Specification

6.5.
A contractor has run air voids tests on five lots of SM-19B. The specification limits for air voids are 4 ± 1.25 %. This sets the lower specification limit (LSL) at 2.75 % (4 - 1.25 %) air voids and the upper specification limit (USL) at 5.25 % (4 + 1.25 %) air voids. Conduct a Quality Level Analysis and compute the percent within limits.

Lot	Sublot	Percent Air Voids
1	1A	4.30
	1B	3.77
	1C	4.05
	1D	4.80
2	2A	4.90
	2B	5.07
	2C	3.82
	2D	3.53

$\bar{x}$ = 4.23%
s = 0.437%
n = 4

LSL = 2.75%
USL = 5.25%

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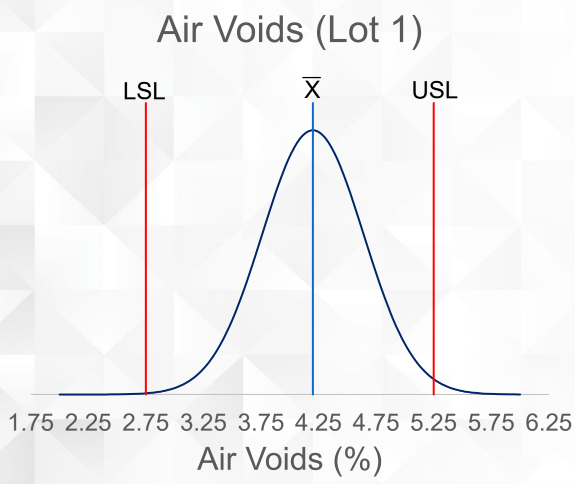
$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$$Q_L = \frac{(4.23 - 2.75)}{0.437}$$

$$Q_L = \frac{(1.48)}{0.437}$$

$$Q_L = 3.39$$



$$Q_U = \frac{(USL - \bar{X})}{s}$$

$$Q_U = \frac{(5.25 - 4.23)}{0.437}$$

$$Q_U = \frac{(1.02)}{0.437}$$

$$Q_U = 2.33$$

$\bar{x}$ = 4.23%
s = 0.437%
n = 4

LSL = 2.75%
USL = 5.25%

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$n = 4$

$Q_L = 3.39$

$PWL_L = 100.00$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
3.15	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.94
3.16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.94
3.17	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.94
3.18	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.94
3.19	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.95
3.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.95
3.21	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.95
3.22	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97	99.95
3.23	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97	99.95
3.24	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.97	99.96
3.25	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.97	99.96
3.26	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.96
3.27	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.96
3.28	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.96
3.29	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96
3.30	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96
3.31	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.32	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.33	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.34	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.35	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.36	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.37	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.38	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.39	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.98
3.40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.41	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.42	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98

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$n = 4$

$Q_U = 2.33$

$PWL_U = 100.00$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
2.25	100.00	100.00	100.00	100.00	100.00	99.91	99.79	99.68	99.34	99.18	99.04	98.93	98.85
2.26	100.00	100.00	100.00	100.00	100.00	99.92	99.80	99.70	99.37	99.21	99.07	98.96	98.88
2.27	100.00	100.00	100.00	100.00	100.00	99.93	99.82	99.71	99.39	99.24	99.10	98.99	98.91
2.28	100.00	100.00	100.00	100.00	100.00	99.94	99.83	99.73	99.42	99.26	99.12	99.02	98.94
2.29	100.00	100.00	100.00	100.00	100.00	99.95	99.85	99.75	99.44	99.29	99.15	99.05	98.97
2.30	100.00	100.00	100.00	100.00	100.00	99.96	99.86	99.77	99.46	99.32	99.18	99.07	99.00
2.31	100.00	100.00	100.00	100.00	100.00	99.96	99.87	99.78	99.48	99.34	99.20	99.10	99.03
2.32	100.00	100.00	100.00	100.00	100.00	99.97	99.89	99.80	99.51	99.36	99.23	99.13	99.05
2.33	100.00	100.00	100.00	100.00	100.00	99.98	99.90	99.81	99.53	99.39	99.25	99.15	99.08
2.34	100.00	100.00	100.00	100.00	100.00	99.98	99.91	99.82	99.55	99.41	99.28	99.18	99.10
2.35	100.00	100.00	100.00	100.00	100.00	99.98	99.92	99.84	99.57	99.43	99.30	99.20	99.13
2.36	100.00	100.00	100.00	100.00	100.00	99.99	99.92	99.85	99.58	99.45	99.32	99.22	99.15
2.37	100.00	100.00	100.00	100.00	100.00	99.99	99.93	99.86	99.60	99.47	99.34	99.25	99.18
2.38	100.00	100.00	100.00	100.00	100.00	99.99	99.94	99.87	99.62	99.49	99.37	99.27	99.20
2.39	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.88	99.64	99.51	99.39	99.29	99.22
2.40	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.89	99.65	99.53	99.41	99.31	99.25
2.41	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.90	99.67	99.55	99.43	99.33	99.27
2.42	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.91	99.68	99.56	99.44	99.35	99.29
2.43	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.92	99.70	99.58	99.46	99.37	99.31
2.44	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.92	99.71	99.60	99.48	99.39	99.33
2.45	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.93	99.73	99.61	99.50	99.41	99.35

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Quality Level Analysis
Percent Within Limits

- For a two-tail specification, the PWL is (Section 6.3.9):

$$PWL = (PWLU + PWL_L) - 100$$

- For Lot 1:

$$PWL = (100 + 100) - 100$$

$$PWL = 100$$

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Quality Level Analysis
Double Limit Specification

6.5.
A contractor has run air voids tests on five lots of SM-19B. The specification limits for air voids are 4 ± 1.25 %. This sets the lower specification limit (LSL) at 2.75 % (4 - 1.25 %) air voids and the upper specification limit (USL) at 5.25 % (4 + 1.25 %) air voids. Conduct a Quality Level Analysis and compute the percent within limits.

Lot	Sublot	Percent Air Voids		
1	1A	4.30	$\bar{x}$ = 4.23%	
	1B	3.77	s = 0.437%	
	1C	4.05	n = 4	
	1D	4.80		
2	2A	4.90	$\bar{x}$ = 4.33%	LSL = 2.75%
	2B	5.07	s = 0.769%	USL = 5.25%
	2C	3.82	n = 4	
	2D	3.53		

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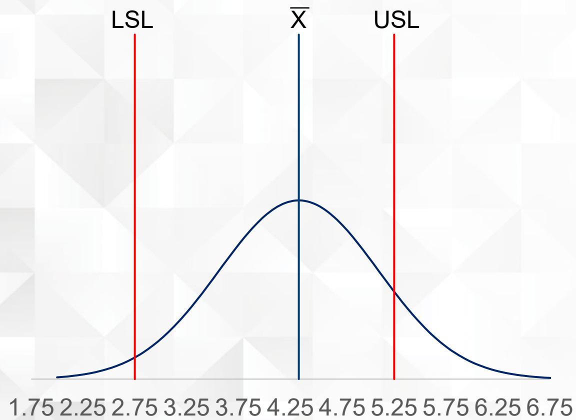
Air Voids (Lot 2)

$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$$Q_L = \frac{(4.33 - 2.75)}{0.769}$$

$$Q_L = \frac{(1.58)}{0.769}$$

$$Q_L = 2.05$$



$$Q_U = \frac{(USL - \bar{X})}{s}$$

$$Q_U = \frac{(5.25 - 4.33)}{0.769}$$

$$Q_U = \frac{(0.92)}{0.769}$$

$$Q_U = 1.20$$

$\bar{x}$ = 4.33%
s = 0.769%
n = 4
LSL = 2.75%
USL = 5.25%

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$n = 4$

$Q_L = 2.05$

$PWL_L = 100.00$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.80	100.00	100.00	100.00	98.99	98.35	97.96	97.70	97.51	97.06	96.87	96.70	96.57	96.49
1.81	100.00	100.00	100.00	99.07	98.43	98.04	97.78	97.60	97.14	96.95	96.78	96.65	96.57
1.82	100.00	100.00	100.00	99.15	98.51	98.12	97.86	97.68	97.21	97.02	96.85	96.73	96.64
1.83	100.00	100.00	100.00	99.22	98.59	98.20	97.94	97.75	97.29	97.10	96.93	96.81	96.72
1.84	100.00	100.00	100.00	99.29	98.66	98.28	98.02	97.83	97.37	97.18	97.01	96.88	96.79
1.85	100.00	100.00	100.00	99.36	98.74	98.35	98.09	97.91	97.44	97.25	97.08	96.95	96.87
1.86	100.00	100.00	100.00	99.43	98.81	98.42	98.16	97.98	97.52	97.32	97.15	97.03	96.94
1.87	100.00	100.00	100.00	99.49	98.88	98.49	98.24	98.05	97.59	97.39	97.22	97.10	97.01
1.88	100.00	100.00	100.00	99.54	98.94	98.56	98.30	98.12	97.66	97.46	97.29	97.17	97.08
1.89	100.00	100.00	100.00	99.60	99.01	98.63	98.37	98.19	97.72	97.53	97.36	97.23	97.15
1.90	100.00	100.00	100.00	99.65	99.07	98.69	98.44	98.25	97.79	97.60	97.43	97.30	97.21
1.91	100.00	100.00	100.00	99.70	99.13	98.76	98.50	98.32	97.86	97.66	97.49	97.37	97.28
1.92	100.00	100.00	100.00	99.74	99.19	98.82	98.56	98.38	97.92	97.73	97.55	97.43	97.34
1.93	100.00	100.00	100.00	99.78	99.24	98.88	98.63	98.44	97.98	97.79	97.62	97.49	97.40
1.94	100.00	100.00	100.00	99.82	99.30	98.93	98.68	98.50	98.04	97.85	97.68	97.55	97.46
1.95	100.00	100.00	100.00	99.85	99.35	98.99	98.74	98.56	98.10	97.91	97.74	97.61	97.52
1.96	100.00	100.00	100.00	99.88	99.40	99.04	98.80	98.62	98.16	97.97	97.80	97.67	97.58
1.97	100.00	100.00	100.00	99.91	99.44	99.09	98.85	98.67	98.22	98.03	97.86	97.73	97.64
1.98	100.00	100.00	100.00	99.93	99.49	99.14	98.90	98.73	98.27	98.08	97.91	97.79	97.70
1.99	100.00	100.00	100.00	99.95	99.53	99.19	98.95	98.78	98.33	98.14	97.97	97.84	97.75
2.00	100.00	100.00	100.00	99.97	99.57	99.24	99.00	98.83	98.38	98.19	98.02	97.90	97.81
2.01	100.00	100.00	100.00	99.98	99.61	99.28	99.05	98.88	98.43	98.24	98.07	97.95	97.86
2.02	100.00	100.00	100.00	99.99	99.64	99.33	99.10	98.93	98.48	98.29	98.13	98.00	97.91
2.03	100.00	100.00	100.00	100.00	99.68	99.37	99.14	98.97	98.53	98.34	98.18	98.05	97.96
2.04	100.00	100.00	100.00	100.00	99.71	99.41	99.18	99.02	98.58	98.39	98.23	98.10	98.01
2.05	100.00	100.00	100.00	100.00	99.74	99.45	99.23	99.06	98.63	98.44	98.27	98.15	98.06

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$n = 4$

$Q_U = 1.20$

$PWL_U = 90.00$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.95	80.75	81.67	82.14	82.39	82.52	82.61	82.67	82.71	82.80	82.84	82.86	82.88	82.89
0.96	81.25	82.00	82.45	82.67	82.80	82.88	82.94	82.97	83.06	83.10	83.12	83.13	83.14
0.97	81.75	82.33	82.75	82.96	83.08	83.15	83.21	83.24	83.32	83.35	83.37	83.39	83.39
0.98	82.26	82.67	83.04	83.24	83.35	83.43	83.47	83.51	83.58	83.61	83.63	83.64	83.64
0.99	82.79	83.00	83.34	83.52	83.63	83.69	83.74	83.77	83.84	83.86	83.88	83.88	83.89
1.00	83.33	83.33	83.64	83.80	83.90	83.96	84.00	84.03	84.09	84.11	84.12	84.13	84.13
1.01	83.89	83.67	83.93	84.08	84.17	84.22	84.26	84.28	84.34	84.36	84.37	84.37	84.38
1.02	84.47	84.00	84.22	84.36	84.44	84.49	84.52	84.54	84.59	84.60	84.61	84.62	84.62
1.03	85.07	84.33	84.52	84.63	84.70	84.75	84.77	84.79	84.83	84.85	84.85	84.85	84.85
1.04	85.69	84.67	84.81	84.91	84.97	85.00	85.03	85.04	85.08	85.09	85.09	85.09	85.09
1.05	86.34	85.00	85.09	85.18	85.23	85.26	85.28	85.29	85.32	85.33	85.33	85.32	85.32
1.06	87.02	85.33	85.38	85.45	85.49	85.51	85.53	85.54	85.56	85.56	85.56	85.55	85.55
1.07	87.73	85.67	85.67	85.71	85.74	85.76	85.78	85.78	85.80	85.80	85.79	85.78	85.78
1.08	88.49	86.00	85.95	85.98	86.00	86.01	86.02	86.03	86.03	86.03	86.02	86.01	86.00
1.09	89.29	86.33	86.24	86.24	86.25	86.26	86.27	86.27	86.26	86.26	86.25	86.23	86.23
1.10	90.16	86.67	86.52	86.50	86.51	86.51	86.51	86.50	86.49	86.48	86.47	86.46	86.45
1.11	91.11	87.00	86.80	86.76	86.75	86.75	86.74	86.74	86.72	86.71	86.69	86.68	86.66
1.12	92.18	87.33	87.07	87.02	87.00	86.99	86.98	86.97	86.95	86.93	86.91	86.89	86.88
1.13	93.40	87.67	87.35	87.28	87.25	87.23	87.21	87.20	87.17	87.15	87.13	87.11	87.09
1.14	94.92	88.00	87.63	87.53	87.49	87.46	87.45	87.43	87.39	87.37	87.34	87.32	87.30
1.15	97.13	88.33	87.90	87.78	87.73	87.70	87.68	87.66	87.61	87.58	87.55	87.53	87.51
1.16	100.00	88.67	88.17	88.03	87.97	87.93	87.90	87.88	87.82	87.79	87.76	87.74	87.72
1.17	100.00	89.00	88.44	88.28	88.21	88.16	88.13	88.10	88.04	88.00	87.97	87.94	87.92
1.18	100.00	89.33	88.71	88.53	88.44	88.39	88.35	88.32	88.25	88.21	88.18	88.15	88.12
1.19	100.00	89.67	88.98	88.77	88.67	88.61	88.57	88.54	88.46	88.42	88.38	88.35	88.32
1.20	100.00	90.00	89.24	89.01	88.90	88.83	88.79	88.76	88.66	88.62	88.58	88.54	88.52
1.21	100.00	90.33	89.50	89.25	89.13	89.06	89.00	88.97	88.87	88.82	88.78	88.74	88.71

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Quality Level Analysis
Percent Within Limits

- For Lot 2:

$PWL = (PWLU + PWL_L) - 100$

$PWL = (90 + 100) - 100$

$PWL = 90$

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Quality Level Analysis
Double Limit Specification

A contractor has run air voids tests on five lots of SM-19B. The specification limits for air voids are 4 ± 1.25 %. This sets the lower specification limit (LSL) at 2.75 % (4 - 1.25 %) air voids and the upper specification limit (USL) at 5.25 % (4 + 1.25 %) air voids. Conduct a Quality Level Analysis and compute the percent within limits.

Lot	Sublot	Percent Air Voids
Extra	E1A	2.30
	E1B	2.54
	E1C	2.86
	E1D	2.74

$\bar{x}$ = 2.61%
s = 0.245%
n = 4

LSL = 2.75%
USL = 5.25%

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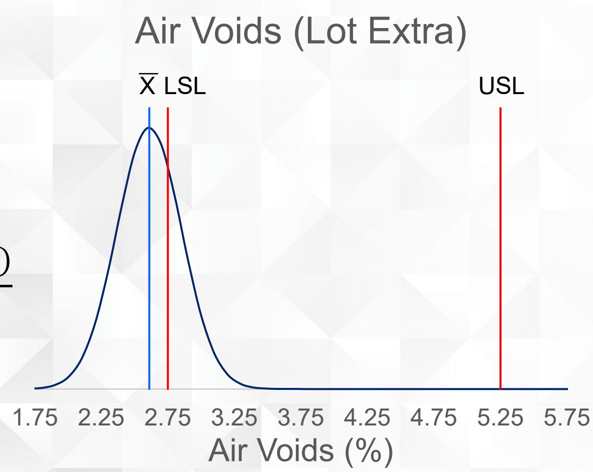
$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$$Q_L = \frac{(2.61 - 2.75)}{0.245}$$

$$Q_L = \frac{(-0.14)}{0.245}$$

Q_L = -0.57



$$Q_U = \frac{(USL - \bar{X})}{s}$$

$$Q_U = \frac{(5.25 - 2.61)}{0.245}$$

$$Q_U = \frac{(2.64)}{0.245}$$

Q_U = 10.78

$\bar{x}$ = 2.61%
s = 0.245%
n = 4
LSL = 2.75%
USL = 5.25%

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$n = 4$

$Q_L = -0.57$

Go into table with:

$Q_L = 0.57$

$PWL_L = 69.00$

for

$Q_L = 0.57$

however,

$Q_L = -0.57$

so,

$PWL_L = 100.00 - 69.00$

$PWL_L = 31.00$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.45	62.74	65.00	65.84	66.27	66.51	66.67	66.79	66.87	67.08	67.16	67.24	67.29	67.33
0.46	63.04	65.33	66.19	66.62	66.87	67.03	67.14	67.22	67.43	67.52	67.60	67.65	67.69
0.47	63.34	65.67	66.53	66.96	67.22	67.38	67.49	67.58	67.79	67.88	67.96	68.01	68.05
0.48	63.65	66.00	66.88	67.31	67.57	67.73	67.85	67.93	68.15	68.23	68.31	68.37	68.40
0.49	63.95	66.33	67.22	67.66	67.92	68.08	68.20	68.28	68.50	68.59	68.67	68.72	68.76
0.50	64.25	66.67	67.56	68.00	68.26	68.43	68.55	68.63	68.85	68.94	69.02	69.07	69.11
0.51	64.56	67.00	67.90	68.35	68.61	68.78	68.90	68.98	69.20	69.29	69.37	69.43	69.46
0.52	64.87	67.33	68.24	68.69	68.96	69.13	69.24	69.33	69.55	69.64	69.72	69.77	69.81
0.53	65.18	67.67	68.58	69.04	69.30	69.47	69.59	69.68	69.90	69.99	70.07	70.12	70.16
0.54	65.49	68.00	68.92	69.38	69.64	69.82	69.93	70.02	70.24	70.33	70.41	70.47	70.51
0.55	65.80	68.33	69.26	69.72	69.99	70.16	70.28	70.36	70.59	70.68	70.76	70.81	70.85
0.56	66.12	68.67	69.60	70.06	70.33	70.50	70.62	70.71	70.93	71.02	71.10	71.15	71.19
0.57	66.43	69.00	69.94	70.40	70.67	70.84	70.96	71.05	71.27	71.36	71.44	71.49	71.53
0.58	66.75	69.33	70.27	70.74	71.01	71.18	71.30	71.39	71.61	71.70	71.78	71.83	71.87
0.59	67.07	69.67	70.61	71.07	71.34	71.52	71.64	71.72	71.95	72.04	72.11	72.17	72.21
0.60	67.39	70.00	70.95	71.41	71.68	71.85	71.97	72.06	72.28	72.37	72.45	72.50	72.54
0.61	67.72	70.33	71.28	71.75	72.02	72.19	72.31	72.40	72.61	72.70	72.78	72.84	72.87
0.62	68.04	70.67	71.61	72.08	72.35	72.52	72.64	72.73	72.95	73.04	73.11	73.17	73.20
0.63	68.37	71.00	71.95	72.41	72.68	72.85	72.97	73.06	73.28	73.37	73.44	73.50	73.53
0.64	68.70	71.33	72.28	72.74	73.01	73.18	73.30	73.39	73.61	73.69	73.77	73.82	73.86
0.65	69.03	71.67	72.61	73.08	73.34	73.51	73.63	73.72	73.93	74.02	74.10	74.15	74.18
0.66	69.37	72.00	72.94	73.40	73.67	73.84	73.96	74.04	74.26	74.34	74.42	74.47	74.51
0.67	69.70	72.33	73.27	73.73	74.00	74.17	74.28	74.37	74.58	74.67	74.74	74.79	74.83
0.68	70.04	72.67	73.60	74.06	74.32	74.49	74.61	74.69	74.90	74.99	75.06	75.11	75.14
0.69	70.39	73.00	73.93	74.39	74.65	74.81	74.93	75.01	75.22	75.30	75.38	75.43	75.46

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$n = 4$

$Q_U = 10.78$

$PWL_U = 100.00$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index	Percent Within Limits for Selected Sample Sizes												
Q_U or Q_L	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
3.60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.61	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.62	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.63	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.64	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.65	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.66	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.67	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.68	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.69	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.71	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.72	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.73	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.74	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.76	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

The estimates of lot percent within limits (PWL) provided in the tables are obtained by numerically integrating the beta distribution function corresponding to Quality Index (Q) and Sample Size (N).

To find PWL from the tables, compute Q from the sample mean and sample standard deviation with unknown population variability, and the lower or upper specification limits.

To find the PWL for a negative Quality Index, first get the PWL for the positive value of the Quality Index from the tables and subtract the result from 100.

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Quality Level Analysis
Percent Within Limits

- For Lot Extra:

$PWL = (PWLU + PWL_L) - 100$

$PWL = (100 + 31) - 100$

$PWL = 31$

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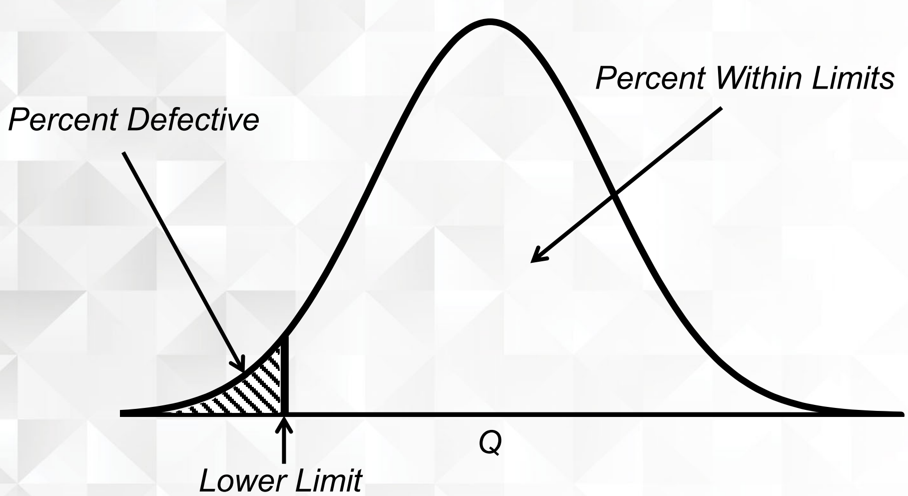
Quality Level Analysis

Single Limit Specification



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Quality Level Analysis
Single Limit Specification

A contractor has made thickness cores on three lots of concrete pavement. The lower specification limit (LSL) is 10.8 in. Conduct a Quality Level Analysis and compute the percent within limits.

Sublot	Thickness (in)
1A	10.9
1B	10.8
1C	10.9
1D	11.0
1E	11.0

$\bar{x}$ = 10.9
s = 0.08
n = 5

LSL = 10.8 in

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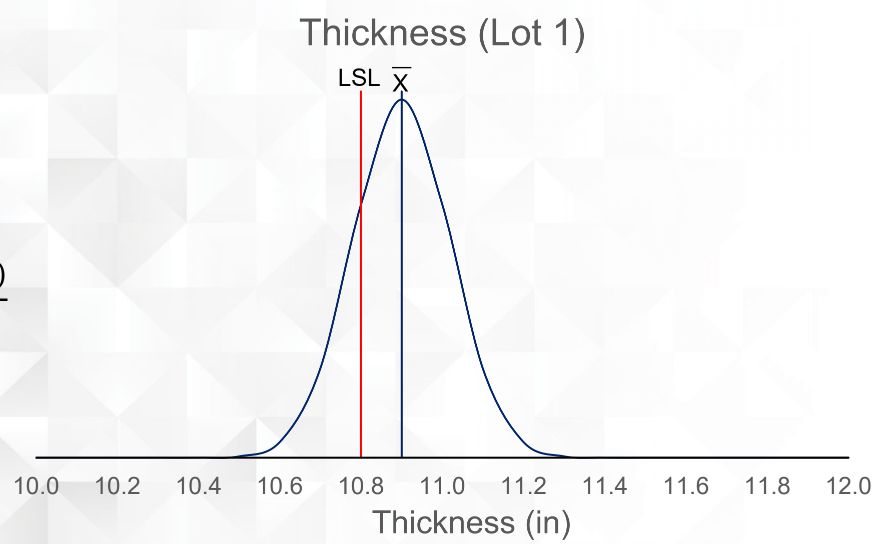
$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$$Q_L = \frac{(10.9 - 10.8)}{0.08}$$

$$Q_L = \frac{(0.1)}{0.08}$$

$$Q_L = 1.25$$



$\bar{x}$ = 10.9 in
s = 0.08
n = 5
LSL = 10.8 in

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Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.00	83.33	83.33	83.64	83.80	83.90	83.96	84.00	84.03	84.09	84.11	84.12	84.13	84.13
1.01	83.89	83.67	83.93	84.08	84.17	84.22	84.26	84.28	84.34	84.36	84.37	84.37	84.38
1.02	84.47	84.00	84.22	84.36	84.44	84.49	84.52	84.54	84.59	84.60	84.61	84.62	84.62
1.03	85.07	84.33	84.52	84.63	84.70	84.75	84.77	84.79	84.83	84.85	84.85	84.85	84.85
1.04	85.69	84.67	84.81	84.91	84.97	85.00	85.03	85.04	85.08	85.09	85.09	85.09	85.09
1.05	86.34	85.00	85.09	85.18	85.23	85.26	85.28	85.29	85.32	85.33	85.33	85.32	85.32
1.06	87.02	85.33	85.38	85.45	85.49	85.51	85.53	85.54	85.56	85.56	85.56	85.55	85.55
1.07	87.73	85.67	85.67	85.71	85.74	85.76	85.78	85.78	85.80	85.80	85.79	85.78	85.78
1.08	88.49	86.00	85.95	85.98	86.00	86.01	86.02	86.03	86.03	86.03	86.02	86.01	86.00
1.09	89.29	86.33	86.24	86.24	86.25	86.26	86.27	86.27	86.26	86.26	86.25	86.23	86.23
1.10	90.16	86.67	86.52	86.50	86.51	86.51	86.51	86.50	86.49	86.48	86.47	86.46	86.45
1.11	91.11	87.00	86.80	86.76	86.75	86.75	86.74	86.74	86.72	86.71	86.69	86.68	86.66
1.12	92.18	87.33	87.07	87.02	87.00	86.99	86.98	86.97	86.95	86.93	86.91	86.89	86.88
1.13	93.40	87.67	87.35	87.28	87.25	87.23	87.21	87.20	87.17	87.15	87.13	87.11	87.09
1.14	94.92	88.00	87.63	87.53	87.49	87.46	87.45	87.43	87.39	87.37	87.34	87.32	87.30
1.15	97.13	88.33	87.90	87.78	87.73	87.70	87.68	87.66	87.61	87.58	87.55	87.53	87.51
1.16	100.00	88.67	88.17	88.03	87.97	87.93	87.90	87.88	87.82	87.79	87.76	87.74	87.72
1.17	100.00	89.00	88.44	88.28	88.21	88.16	88.13	88.10	88.04	88.00	87.97	87.94	87.92
1.18	100.00	89.33	88.71	88.53	88.44	88.39	88.35	88.32	88.25	88.21	88.18	88.15	88.12
1.19	100.00	89.67	88.98	88.77	88.67	88.61	88.57	88.54	88.46	88.42	88.38	88.35	88.32
1.20	100.00	90.00	89.24	89.01	88.90	88.83	88.79	88.76	88.66	88.62	88.58	88.54	88.52
1.21	100.00	90.33	89.50	89.25	89.13	89.06	89.00	88.97	88.87	88.82	88.78	88.74	88.71
1.22	100.00	90.67	89.77	89.49	89.35	89.27	89.22	89.18	89.07	89.02	88.97	88.93	88.91
1.23	100.00	91.00	90.03	89.72	89.58	89.49	89.43	89.39	89.27	89.22	89.16	89.12	89.09
1.24	100.00	91.33	90.28	89.96	89.80	89.70	89.64	89.59	89.47	89.41	89.36	89.31	89.28
1.25	100.00	91.67	90.54	90.19	90.02	89.91	89.85	89.79	89.66	89.60	89.54	89.50	89.47
1.26	100.00	92.00	90.79	90.42	90.23	90.12	90.05	90.00	89.85	89.79	89.73	89.68	89.65
1.27	100.00	92.33	91.04	90.64	90.45	90.33	90.25	90.19	90.04	89.98	89.91	89.87	89.83

$n = 5$

$Q_L = 1.25$

$PWL_L = 90.54$

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Quality Level Analysis
Single Limit Specification

A contractor has made thickness cores on three lots of concrete pavement. The lower specification limit (LSL) is 10.8 in. Conduct a Quality Level Analysis and compute the percent within limits.

Sublot	Thickness (in)
2A	11.4
2B	10.9
2C	10.8
2D	10.2
2E	10.8

$\bar{x}$ = 10.8
s = 0.45
n = 5

LSL = 10.8 in

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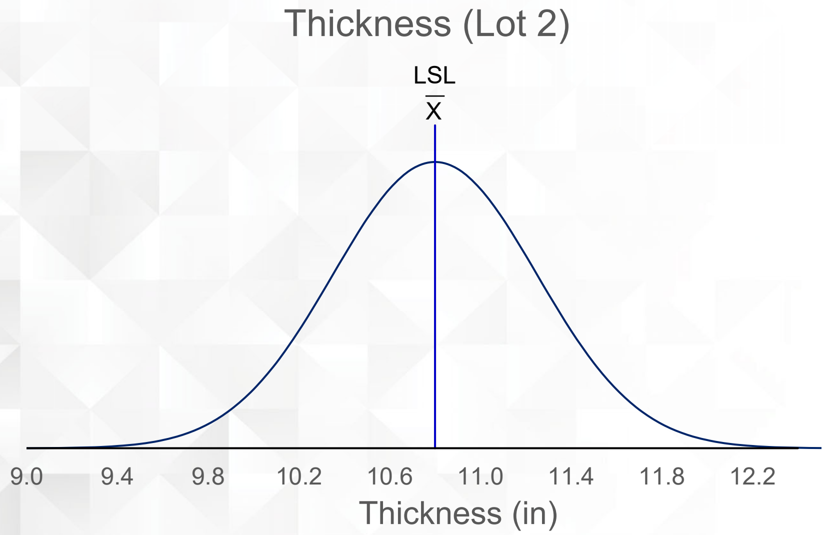
Thickness (Lot 2)

$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$$Q_L = \frac{(10.8 - 10.8)}{0.45}$$

$$Q_L = \frac{(0.0)}{0.45}$$

$$Q_L = 0.00$$



$\bar{x}$ = 10.8 in LSL = 10.8 in
s = 0.45
n = 5

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Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
0.01	50.28	50.33	50.36	50.37	50.37	50.38	50.38	50.38	50.39	50.39	50.40	50.40	50.40
0.02	50.55	50.67	50.71	50.73	50.75	50.76	50.76	50.77	50.78	50.79	50.79	50.79	50.80
0.03	50.83	51.00	51.07	51.10	51.12	51.14	51.15	51.15	51.17	51.18	51.19	51.19	51.19
0.04	51.10	51.33	51.42	51.47	51.50	51.51	51.53	51.54	51.56	51.57	51.58	51.59	51.59
0.05	51.38	51.67	51.78	51.84	51.87	51.89	51.91	51.92	51.95	51.96	51.98	51.98	51.99
0.06	51.65	52.00	52.13	52.20	52.24	52.27	52.29	52.30	52.34	52.36	52.37	52.38	52.39
0.07	51.93	52.33	52.49	52.57	52.62	52.65	52.67	52.69	52.73	52.75	52.76	52.78	52.78
0.08	52.21	52.67	52.85	52.94	52.99	53.03	53.05	53.07	53.12	53.14	53.16	53.17	53.18
0.09	52.48	53.00	53.20	53.30	53.37	53.41	53.43	53.46	53.51	53.53	53.55	53.57	53.58
0.10	52.76	53.33	53.56	53.67	53.74	53.78	53.82	53.84	53.90	53.92	53.95	53.96	53.97
0.11	53.04	53.67	53.91	54.04	54.11	54.16	54.20	54.22	54.29	54.31	54.34	54.36	54.37
0.12	53.31	54.00	54.27	54.40	54.49	54.54	54.58	54.60	54.67	54.70	54.73	54.75	54.76
0.13	53.59	54.33	54.62	54.77	54.86	54.92	54.96	54.99	55.06	55.09	55.12	55.14	55.16
0.14	53.87	54.67	54.98	55.14	55.23	55.29	55.34	55.37	55.45	55.48	55.52	55.54	55.55
0.15	54.15	55.00	55.33	55.50	55.60	55.67	55.71	55.75	55.84	55.87	55.91	55.93	55.95
0.16	54.42	55.33	55.69	55.87	55.97	56.04	56.09	56.13	56.22	56.26	56.30	56.32	56.34
0.17	54.70	55.67	56.04	56.23	56.35	56.42	56.47	56.51	56.61	56.65	56.69	56.71	56.73
0.18	54.98	56.00	56.40	56.60	56.72	56.79	56.85	56.89	56.99	57.04	57.08	57.11	57.12
0.19	55.26	56.33	56.75	56.96	57.09	57.17	57.23	57.27	57.38	57.43	57.47	57.50	57.52
0.20	55.54	56.67	57.10	57.32	57.46	57.54	57.60	57.65	57.76	57.81	57.85	57.89	57.91
0.21	55.82	57.00	57.46	57.69	57.83	57.92	57.98	58.03	58.15	58.20	58.24	58.27	58.30
0.22	56.10	57.33	57.81	58.05	58.20	58.29	58.36	58.40	58.53	58.58	58.63	58.66	58.69
0.23	56.38	57.67	58.16	58.41	58.56	58.66	58.73	58.78	58.91	58.97	59.01	59.05	59.07
0.24	56.66	58.00	58.52	58.78	58.93	59.03	59.11	59.16	59.29	59.35	59.40	59.44	59.46
0.25	56.95	58.33	58.87	59.14	59.30	59.41	59.48	59.53	59.67	59.73	59.78	59.82	59.85
0.26	57.23	58.67	59.22	59.50	59.67	59.78	59.85	59.91	60.05	60.11	60.17	60.21	60.23
0.27	57.51	59.00	59.57	59.86	60.03	60.15	60.23	60.28	60.43	60.49	60.55	60.59	60.62
0.28	57.80	59.33	59.92	60.22	60.40	60.52	60.60	60.66	60.81	60.87	60.93	60.97	61.00
0.29	58.08	59.67	60.28	60.58	60.77	60.89	60.97	61.03	61.19	61.25	61.31	61.35	61.38

$n = 5$

$Q_L = 0.00$

$PWL_L = 50.00$

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Quality Level Analysis
Objectives

- Define Quality Level Analysis
- State and calculate upper and lower quality index formulas
- Use a table to find PWL
- Select and apply proper analyses

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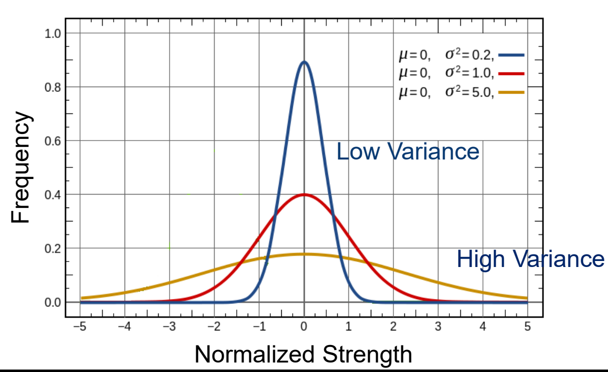
(4) Comparison Testing

- Companion cores will be measured and tested by KDOT for each lot
  - Variances (spread in data) will be compared (F-test)



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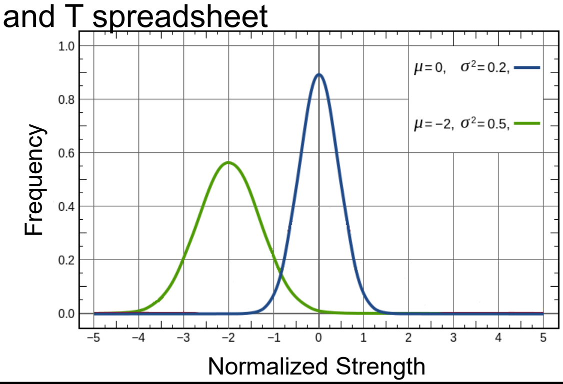
(4) Comparison Testing

- Companion cores will be measured and tested by KDOT for each lot
  - Variances (spread in data) will be compared (F-test)
  - Averages will be compared (T-test)
  - F and T spreadsheet



Do these samples come from the same population?

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(4) Comparison Testing

- Pavement Acceptance and Pay Adjustments
  - Based on Contractor QC results
    - Provided Statistical Comparison is Favorable
    - Random Samples taken from Lot
  - Based on KDOT Verification results
    - When Statistical Comparison is not Favorable
    - Random Sample from the Lot

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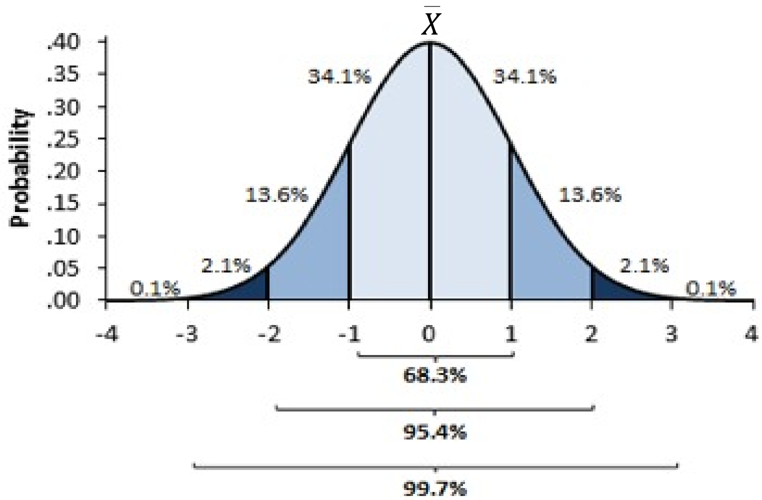
Quality Level Analysis



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COMPARISON TESTING
F-TEST

KDOT Construction Manual

Section 5.2.6

Pages 1-4

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1

Comparison Testing
Objectives

- Understand the need to compare test results
- Understand level of significance (α)
- Understand test comparison procedure
- Learn and execute **F-test** method

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Comparison Testing
Why?

- Compare QC test results and KDOT verification test results.
- Determine if the material under test came from the same population.
- Use **F-test** (Fisher's exact test) and **t-test** (Student's t-test) to compare test results.

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Comparison Testing
Level of Significance (α)

- F-test and t-test are conducted with a selected level of significance (α)
- Level of Significance (α) used is 1%
  - » 1% chance that test results are incorrect
    1) good material/product will be rejected
    2) bad material/ product will be accepted
  - » 99% level of confidence that F-test and t-test results are correct

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Comparison Testing
Procedure

1) Use F-test to compare QC and KDOT variances
2) Use F-test result to determine which t-test method to use
3) Use t-test to compare QC and KDOT means
4) Use t-test result to determine if QC and KDOT test results are statistically equal

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F-TEST

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F-test
Compare Variances

- F-test determines if QC variance (s_c^2) and KDOT variance (s_v^2) are statistically equal or not:

If $F \geq F_{CRIT}$, then variances **not** equal.
OR
If $F < F_{CRIT}$, then variances **are** equal.

- F-test determines what t-test procedure to use.

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F-test
Procedure

i. Find basic statistics for both contractor and KDOT:

n

$\bar{x}$

s

s^2

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F-test
Procedure

ii. Compute F-statistic value:

$$F = \frac{s_c^2}{s_v^2} \text{ or } \frac{s_v^2}{s_c^2}$$

Note: Larger variance always goes in numerator (top)

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F-test
Procedure

iii. Choose $\alpha = 1\%$ level of significance

$$F = \frac{s_c^2}{s_v^2} \text{ or } \frac{s_v^2}{s_c^2}$$

iv. Degrees of Freedom

Numerator: $n_{num} - 1$ *Larger Variance*

Denominator: $n_{den} - 1$

Note: Table 1 is for a *two-tailed test* – Difference between 2 variance estimates

A *one-tailed test* – if one variance is larger than another

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F-test Procedure

v. Find F_{CRIT} in Table 1 (Pages 3-4)

Table 1 **Critical Values, F_{crit} for the F-test for a Level of Significance, α =1%**

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

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Table 1 **Critical Values, F_{crit} , for the F-test for a Level of Significance, α =1 % (contd..)**

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	15	20	24	30	40	50	60	100	120	200	500	∞
1	24600	24800	24900	25000	25100	25200	25300	25300	25400	25400	25400	25500
2	199	199	199	199	199	199	199	199	199	199	199	200
3	43.1	42.8	42.69	42.5	42.3	42.2	42.1	42.0	42.0	41.9	41.9	41.8
4	20.4	20.2	20.0	19.9	19.8	19.7	19.6	19.5	19.5	19.4	19.4	19.3
5	13.1	12.9	12.8	12.7	12.5	12.5	12.4	12.3	12.3	12.2	12.2	12.1
6	9.81	9.59	9.47	9.36	9.24	9.17	9.12	9.03	9.00	8.95	8.91	8.88
7	7.97	7.75	7.65	7.53	7.42	7.35	7.31	7.22	7.19	7.15	7.10	7.08
8	6.81	6.61	6.50	6.40	6.29	6.22	6.18	6.09	6.06	6.02	5.98	5.95
9	6.03	5.83	5.73	5.62	5.52	5.45	5.41	5.32	5.30	5.26	5.21	5.19
10	5.47	5.27	5.17	5.07	4.97	4.90	4.86	4.77	4.75	4.71	4.67	4.64
11	5.05	4.86	4.76	4.65	4.55	4.49	4.45	4.36	4.34	4.29	4.25	4.23
12	4.72	4.53	4.43	4.33	4.23	4.17	4.12	4.04	4.01	3.97	3.93	3.90
15	4.07	3.88	3.79	3.69	3.59	3.52	3.48	3.39	3.37	3.33	3.29	3.26
20	3.50	3.32	3.22	3.12	3.02	2.96	2.92	2.83	2.81	2.76	2.72	2.69
24	3.25	3.06	2.97	2.87	2.77	2.70	2.66	2.57	2.55	2.50	2.46	2.43
30	3.01	2.82	2.73	2.63	2.52	2.46	2.42	2.32	2.30	2.25	2.21	2.18
40	2.78	2.60	2.50	2.40	2.3	2.23	2.18	2.09	2.06	2.01	1.96	1.93
60	2.57	2.39	2.29	2.19	2.08	2.01	1.96	1.86	1.83	1.78	1.73	1.69
120	2.37	2.19	2.09	1.98	1.87	1.80	1.75	1.64	1.61	1.54	1.48	1.43
∞	2.19	2.00	1.90	1.79	1.67	1.59	1.53	1.40	1.36	1.28	1.17	1.00

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

$H_o: \; s^2_c = s^2_v$

$H_a: \; s^2_c \neq s^2_v$

S

tion

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F-test
Procedure

vi. Compare calculated F value and F_{CRIT}

Two conclusions:

If $F \geq F_{CRIT}$, then variances **not** equal.

OR

If $F < F_{CRIT}$, then variances **are** equal.

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EXAMPLE PROBLEM
Case 1 - Concrete

Page 5.2.6-8

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Example Problem – Case 1 (Page 8)

A contractor has run 21 QC tests for compressive strength and KDOT has run 5 verification tests over the same period of time. The results are shown below. Is it likely that the tests came from the same population?

Contractor QC Test Results (%)	KDOT VerificationTest Results (%)
36.40	36.10
36.65	30.00
32.69	37.00
38.05	32.80
38.54	30.60
37.59	
36.57	
42.48	
36.99	
38.20	
37.53	
36.00	
41.28	
40.00	
38.37	
38.72	
40.36	
30.37	
34.87	
35.62	
36.06	

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Example Problem – Case 1

Step 1: Compute the Statistics

Contractor	**KDOT**
$n_c = 21$	$n_v = 5$
$\bar{x}_c = 37.302$	$\bar{x}_v = 33.300$
$s_c = 2.736$	$s_v = 3.161$
${s_c}^2 = 7.431$	${s_v}^2 = 9.992$

Step 2: Compute the F Statistic

$$F = \frac{{s_c}^2}{{s_v}^2} \text{ or } \frac{{s_v}^2}{{s_c}^2}$$

$$F = \frac{9.992}{7.431} = 1.34$$

Note: Larger variance always goes in numerator (top)

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Example Problem – Case 1
Step 3 and 4

3. Level of Significance: $\alpha = 1\%$
4. Find the critical F Value (F_{CRIT})

Degrees of Freedom

- Numerator (Larger Variance):

  KDOT's Variance is 9.992 is the larger value

- Denominator (Smaller Variance):

  Contractor Variance is 7.431 is the smaller value

Therefore, KDOT's degree of Freedom is the Numerator

$$df_{num} = n_{num} - 1 : \quad n_v = 5, \text{ thus } df_{num} = 5 - 1 = 4$$

$$df_{den} = n_{den} - 1 : \quad n_c = 21, \text{ thus } df_{den} = 21 - 1 = 20$$

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Example Problem – Case 1
F_{CRIT}

Table 1 **Critical Values, F_{crit} for the F-test for a Level of Significance, α =1%**

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

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Example Problem – Case 1

Contractor	KDOT
$n_c = 21$	$n_v = 5$
$\bar{x}_c = 37.302$	$\bar{x}_v = 33.300$
$s_c = 2.736$	$s_v = 3.161$
$s_c{}^2 = 7.431$	$s_v{}^2 = 9.992$
$\mathrm{df}_{den} = 20$	$\mathrm{df}_{num} = 4$

$F = 1.34$

$F_{CRIT} = 5.17$

If $F \geq F_{CRIT}$ then variances **not** equal.

OR

If $F < F_{CRIT}$ then variances **are** equal.

Since F (1.34) is less than F_{CRIT} (5.17), then the variances are statistically equal

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Example Problem – Case 1

Conclusion from the F-test

$F = 1.34$ $F_{CRIT} = 5.17$

- Since $F < F_{CRIT}$ assume the sample variances are statistically equal
- Use the ***pooled estimate for the variance*** and the ***pooled degrees of freedom*** when conducting the t-test

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Example Problem – Case 2 (Page 10)

A contractor has run 10 QC tests and KDOT has run 5 verification tests over the same period of time for the asphalt pavement density (%G_{mm}). The results are shown below. Is it likely that the test came from the same population or lot?

Contractor QC Test Results	KDOT Verification Test Results
93.0	95.5
92.4	93.3
92.9	94.1
93.6	92.5
92.9	92.7
92.9	
92.4	
93.4	
92.9	
92.4	

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Example Problem – Case 2
Step 1: Compute the Statistics

Contractor	**KDOT**
$n_c = 10$	$n_v = 5$
$\bar{x}_c = 92.88$	$\bar{x}_v = 93.62$
$s_c = 0.408$	$s_v = 1.221$
$s_c{}^2 = 0.166$	$s_v{}^2 = 1.491$

Step 2: Compute the F Statistic

$$F = \frac{s_c^2}{s_v^2} \text{ or } \frac{s_v^2}{s_c^2}$$

$$F = \frac{1.491}{0.166} = 8.98$$

Note: Larger variance always goes in numerator (top)

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Example Problem – Case 2

Step 3 and 4

3. Level of Significance: $\alpha = 1\%$
4. Find the critical F Value (F_{CRIT})

Degrees of Freedom

- Numerator (Larger Variance):

KDOT's Variance is 1.491 is the larger value

- Denominator (Smaller Variance):

Contractor Variance is 0.166 is the smaller value

Therefore, KDOT's degree of Freedom is the Numerator

$\mathrm{df}_{num} = n_{num} - 1: \quad n_v = 5, \text{ thus } \mathrm{df}_{num} = 5 - 1 = 4$

$\mathrm{df}_{den} = n_{den} - 1: \quad n_c = 10, \text{ thus } \mathrm{df}_{den} = 10 - 1 = 9$

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Example Problem – Case 2

F_{CRIT}

Table 1 **Critical Values, F_{crit} for the F-test for a Level of Significance, α =1%**

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

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Example Problem – Case 2

Contractor	**KDOT**
$n_c = 10$	$n_v = 5$
$\bar{x}_c = 92.88$	$\bar{x}_v = 93.62$
$s_c = 0.408$	$s_v = 1.221$
$s_c^2 = 0.166$	$s_v^2 = 1.491$
$\text{df}_{den} = 9$	$\text{df}_{num} = 4$

$F = 8.98$

$F_{CRIT} = 7.96$

If $F \geq F_{CRIT}$ then variances **not** equal.
OR
If $F < F_{CRIT}$ then variances **are** equal.

Since *F* (8.98) is greater than F_{CRIT} (7.96), then the variances are not statistically equal

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Example Problem – Case 2

Conclusion from the F-test

$F = 8.98$ $F_{CRIT} = 7.96$

- Since $F \geq F_{CRIT}$, assume the variances are not statistically equal
- Use ***individual sample variances, individual sample sizes,*** and the ***effective degrees of freedom (estimated from the sample variances and sample sizes)*** when conducting the t-test

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Takeaways

- F-test compares variances s_c^2 and s_v^2
- There is only one F-test procedure
- F-test result only tells us which t-test procedure to use
  - Variances are equal
  - Variances not equal

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Comparison Testing
Objectives

- Understand the need to compare test results
- Understand level of significance (α)
- Understand test comparison procedure
- Learn and execute **F-test** method

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COMPARISON TESTING
t-test

KDOT Construction Manual

Section 5.2.6

Pages 5-12

1

Comparison Testing
Objectives

- Learn and execute the **t-test** for equal variances
- Learn and execute the **t-test** for not equal variances

2

Comparison Testing
Procedure

1) Use F-test to compare QC and KDOT variances
2) Use F-test result to determine which t-test method to use
3) Use t-test to compare QC and KDOT means
4) Use t-test result to determine if QC and KDOT test results are statistically equal

3

t-test
Variances are Equal

4

Case 1 – Sample Variance Assumed to Be Equal Page 5 (5.2.6.)

Case 1: **Sample Variances Assumed to Be Equal**

a) To conduct the t-test when the sample variances are assumed equal, Equation 1 is used to calculate the t value from which the decision is reached.

$$t = \frac{\left|\overline{X}_c - \overline{X}_v\right|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_v}}} \qquad (1)$$

where:
- $\overline{X}_c$ = mean of QC tests
- $\overline{X}_v$ = mean of verification tests
- s_p^2 = pooled estimate for the variance (described below)
- n_c = number of QC tests
- n_v = number of verification tests

b) The pooled variance, which is the weighted average, using the degrees of freedom for each sample as the weighting factor, is computed from the sample variances using Equation 2.

$$s_p^2 = \frac{s_c^2(n_c - 1) + s_v^2(n_v - 1)}{n_c + n_v - 2} \qquad (2)$$

Where:
- s_p^2 = pooled estimate for the variance
- n_c = number of QC tests
- n_v = number of verification tests
- s_c^2 = variance of the QC tests
- s_v^2 = variance of the verification tests

c) Once the pooled variance is estimated, the value of t is computed using equation 1.

d) To determine the critical t value against which to compare the computed t value, it is necessary to select the level of significance, α. *A value of $\alpha = 1$ % is recommended.*

e) Determine the critical t value, t_{crit}, from **Table 2** for the pooled degrees of freedom. The pooled degrees of freedom for the case where the sample variances are assumed equal is $(n_c + n_v - 2)$.

f) If $t \geq t_{crit}$, then decide that the two sets of tests have significantly different means. If $t < t_{crit}$, then decide that there is no reason to believe that the means are significantly different.

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t-test

Compare Means

- t-test determines if the QC mean ($\overline{X}_c$) and KDOT mean ($\overline{X}_v$) are equal or not:

If $t \geq t_{CRIT}$, then means **not** equal.

OR

If $t < t_{CRIT}$, then means **are** equal.

- **If there are no differences in the sample means, it indicates the material under test came from the same population**

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Use pooled variance (s_p^2), which is the weighted average of variance from both sets of tests when calculating t value
- Use pooled degrees of freedom ($n_c + n_v - 2$)
- When $t = 0$, the two data sets have exactly the same means

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Calculate pooled variance s_p^2

$$s_p^2 = \frac{s_c^2(n_c - 1) + s_v^2(n_v - 1)}{n_c + n_v - 2}$$

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Calculate t statistic

$$t = \frac{\left|\overline{X}_c - \overline{X}_v\right|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_v}}}$$

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t-test

Variances **are** equal ($F < F_{CRIT}$)

- Determine $\alpha = 1\%$
- Calculate degrees of freedom

$$df = n_c + n_v - 2$$

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t-test

Variances **are** equal ($F < F_{CRIT}$)

Find t_{CRIT} in Table 2
Page 7

degrees of freedom	α = 0.01	α = 0.05	α = 0.10
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708

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t-test

Variances **are** equal ($F < F_{CRIT}$)

Compare calculated t value and t_{CRIT}

Two conclusions:

If $t \geq t_{CRIT}$, then means **not** equal.

OR

If $t < t_{CRIT}$, then means **are** equal.

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EXAMPLE PROBLEM
Case 1 – Concrete
(continued)

Pg. 5.2.6-8

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Example Problem – Case 1

Contractor	**KDOT**	
$n_c = 21$	$n_v = 5$	$F = 1.34$
$\bar{x}_c = 37.302$	$\bar{x}_v = 33.300$	$F_{CRIT} = 5.17$
$s_c = 2.736$	$s_v = 3.161$	$F < F_{CRIT}$
${s_c}^2 = 7.431$	${s_v}^2 = 9.992$	
$\text{df}_{den} = 20$	$\text{df}_{num} = 4$	

Calculate pooled variance $s_p^2 = \dfrac{s_c^2(n_c - 1) + s_v^2(n_v - 1)}{n_c + n_v - 2}$

$$ {s_p}^2 = \frac{7.431\ (21 - 1) + 9.992\ (5 - 1)}{21 + 5\ - 2} = \frac{7.431\ (20) + 9.992\ (4)}{24} $$

$$ {s_p}^2 = \frac{148.62 + 39.968}{24} = \frac{188.588}{24} \qquad \boldsymbol{s_p}^2 = \mathbf{7.858} $$

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Example Problem – Case 1
Determining the t-statistic

Contractor

$n_c = 21$
$\bar{x}_c = 37.302$
$s_c = 2.736$
$s_c^2 = 7.431$
$df_{den} = 20$

KDOT

$n_v = 5$
$\bar{x}_v = 33.300$
$s_v = 3.161$
$s_v^2 = 9.992$
$df_{num} = 4$

$F = 1.34$
$F_{CRIT} = 5.17$
$F < F_{CRIT}$

$s_p^2 = 7.858$

$$t = \frac{|\bar{X}_c - \bar{X}_v|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_v}}}$$

Calculate t-statistic, t

$$t = \frac{|37.302 - 33.300|}{\sqrt{\frac{7.858}{21} + \frac{7.858}{5}}} = \frac{|4.002|}{\sqrt{0.374 + 1.572}} = \frac{4.002}{\sqrt{1.946}} = \frac{4.002}{1.395} = 2.869$$

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Example Problem – Case 1
Determining the t-statistic

Contractor

$n_c = 21$
$\bar{x}_c = 37.302$
$s_c = 2.736$
$s_c^2 = 7.431$
$df_{den} = 20$

KDOT

$n_v = 5$
$\bar{x}_v = 33.300$
$s_v = 3.161$
$s_v^2 = 9.992$
$df_{num} = 4$

$F = 1.34$
$F_{CRIT} = 5.17$
$F < F_{CRIT}$

$s_p^2 = 7.858$
$t = 2.869$

Calculate critical t-value, t_{crit}
Pooled Degrees of Freedom = $n_c + n_v - 2 = 21 + 5 - 2 = 24$
$\alpha = 1\%$
Enter Table 2 with the Pooled Degrees of Freedom and α

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t_{CRIT}

degrees of freedom	α = 0.01	α = 0.05	α = 0.10
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

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Example Problem – Case 1

t = 2.869 t_{crit} = 2.797

$t \geq t_{crit}$

Thus, we assume that the sample means are not equal and the two sample sets did not come from the same population

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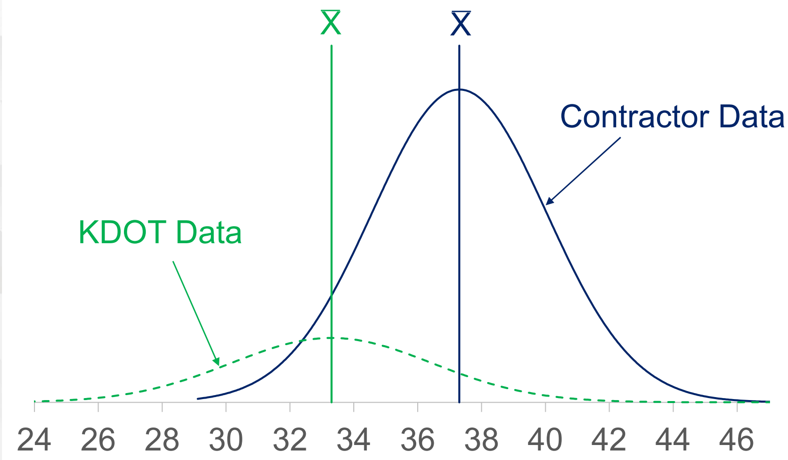
Example Problem – Case 1



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t-test
Variances not Equal

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Case 2 – Sample Variances Assumed to Be Not Equal
Page 6 (5.2.6.)

Case 2: **Sample Variances Assumed to Be Not Equal**

a) To conduct the t-test when the sample variances are assumed not equal, Equation 3 is used to calculate the t value from which the decision is reached.

$$t = \frac{\left|\bar{X}_c - \bar{X}_v\right|}{\sqrt{\frac{s_c^2}{n_c} + \frac{s_v^2}{n_v}}} \qquad (3)$$

where:

$\bar{X}_c$ = mean of QC tests
$\bar{X}_v$ = mean of verification tests
s_c^2 = variance of the QC tests
s_v^2 = variance of the verification tests
n_c = number of QC tests
n_v = number of verification tests

b) To determine the critical t value against which to compare the computed t value, it is necessary to select the level of significance, α. A value of α = 1% is recommended.

c) The effective degrees of freedom, f', for the case where the sample variances are assumed not equal is determined from Equation 4 *(the calculated effective degrees of freedom is rounded down to a whole number).*

$$f' = \frac{\left(\frac{s_c^2}{n_c} + \frac{s_v^2}{n_v}\right)^2}{\left(\frac{\left(\frac{s_c^2}{n_c}\right)^2}{n_c + 1} + \frac{\left(\frac{s_v^2}{n_v}\right)^2}{n_v + 1}\right)} - 2 \qquad (4)$$

where all the symbols are as described previously.

d) Determine the critical t value, t_{crit}, from **Table 2** for the effective degrees of freedom determined by Equation 4.

e) If $t \geq t_{crit}$, then decide that the two sets of tests have significantly different means. If $t < t_{crit}$, then decide that there is no reason to believe that the means are significantly different.

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t-test

Variances **not** equal ($F \geq F_{CRIT}$)

- Use individual variances from test results to calculate t statistic
- Calculate effective degrees of freedom (f'), effective degrees of freedom rounded **down** to a whole number
- When $t = 0$, the two data sets have exactly the same means

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t-test

Variances **not** equal ($F \geq F_{CRIT}$)

- Use individual variances from test results to calculate t statistic
  - No s_p^2 to calculate
- Effective degrees of freedom (f')
  - Will be provided
- When $t = 0$, the two data sets have exactly the same means

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t-test

Variances **not** equal ($F \geq F_{CRIT}$)

- Determine $\alpha = 1\%$
- Calculate *effective* degrees of freedom f' (given!)

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t-test

Variances **not** equal ($F \geq F_{CRIT}$)

Find t_{CRIT} in Table 2
Page 7

Table 2 **Critical t values**

degrees of freedom	α = 0.01	α = 0.05	α = 0.10
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

NOTE : This is for a two-tailed test with the null and alternate hypotheses shown below :

$H_o: \quad \bar{X}_c = \bar{X}_v$

$H_a: \quad \bar{X} \neq \bar{X}$

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t-test

Variances **not** equal ($F \geq F_{CRIT}$)

Compare calculated t value and t_{CRIT}

Two conclusions:

If $t \geq t_{CRIT}$, then means **not** equal.

OR

If $t < t_{CRIT}$, then means **are** equal.

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EXAMPLE PROBLEM
Case 2 – Asphalt (continued)

Pg. 5.2.6-10

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Example Problem – Case 2
Determining the t-statistic

Contractor	**KDOT**	
$n_c = 10$	$n_v = 5$	$F = 8.98$
$\bar{x}_c = 92.88$	$\bar{x}_v = 93.62$	$F_{CRIT} = 7.96$
$s_c = 0.408$	$s_v = 1.221$	$F \geq F_{CRIT}$
${s_c}^2 = 0.166$	${s_v}^2 = 1.491$	
$\mathrm{df}_{den} = 9$	$\mathrm{df}_{num} = 4$	

$$t = \frac{|\bar{X}_c - \bar{X}_v|}{\sqrt{\frac{s_c^2}{n_c} + \frac{s_v^2}{n_v}}}$$

Calculate t-statistic, t

$$t = \frac{|\,92.88 - 93.62\,|}{\sqrt{\frac{0.166}{10} + \frac{1.491}{5}}} = \frac{|\,-0.74\,|}{\sqrt{0.0166 + 0.2982}} = \frac{0.74}{\sqrt{0.3148}} = \frac{0.74}{0.561} = 1.319$$

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Example Problem – Case 2
Calculate the effective degrees of freedom

Contractor	**KDOT**		
$n_c = 10$	$n_v = 5$	$F = 8.98$	$\boldsymbol{t} = 1.319$
$\bar{x}_c = 92.88$	$\bar{x}_v = 93.62$	$F_{CRIT} = 7.96$	
$s_c = 0.408$	$s_v = 1.221$		
${s_c}^2 = 0.166$	${s_v}^2 = 1.491$	$F \geq F_{CRIT}$	
$\mathrm{df}_{den} = 9$	$\mathrm{df}_{num} = 4$		

$$f' = \frac{\left(\frac{s_c^2}{n_c} + \frac{s_v^2}{n_v}\right)^2}{\left(\frac{\left(\frac{s_c^2}{n_c}\right)^2}{n_c + 1} + \frac{\left(\frac{s_v^2}{n_v}\right)^2}{n_v + 1}\right)} - 2$$

Calculate effective degrees of freedom

$f' = 4.68$

$f' = 4$

Round **down** to a whole number

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Example Problem – Case 2
Determining the t-statistic

Contractor	**KDOT**		
$n_c = 10$	$n_v = 5$	$F = 8.98$	$\boldsymbol{t} = 1.319$
$\bar{x}_c = 92.88$	$\bar{x}_v = 93.62$	$F_{CRIT} = 7.96$	$\boldsymbol{f'} = 4$
$s_c = 0.408$	$s_v = 1.221$	$F \geq F_{CRIT}$	
${s_c}^2 = 0.166$	${s_v}^2 = 1.491$		
$\mathrm{df}_{den} = 9$	$\mathrm{df}_{num} = 4$		

Calculate critical t-value, t_{crit}
Degrees of Freedom = 4
$\alpha = 1\%$
Enter Table 2 with the Degrees of Freedom and α

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t_{CRIT}

degrees of freedom	α = 0.01	α = 0.05	α = 0.10
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
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22	2.819	2.074	1.717
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26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

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Example Problem – Case 2

t = 1.319 t_{crit} = 4.604

$t < t_{crit}$

Thus, we assume that the sample means are equal and the two sample sets did come from the same population

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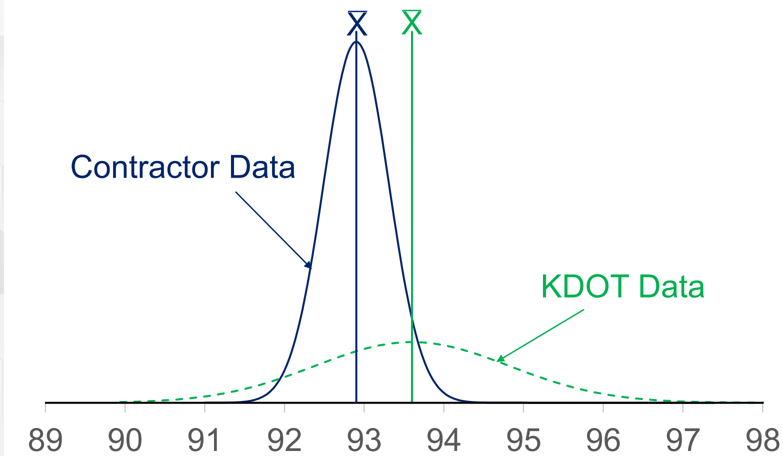
Example Problem – Case 2



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Example Problem – Case 2

Conclusion for t-test

t = 1.319 $t_{CRIT} = 4.604$

- Since $t < t_{CRIT}$, assume the sample means are equal
- Even though variances were assumed not equal as result of **F-test**, the **t-test** indicates that the sets of tests results came from populations (lots) that had the same mean

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Comparison Testing
Objectives

- Learn and execute the **t-test** for variances are equal
- Learn and execute the **t-test** for variances not equal

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Takeaways

- Comparison testing is a two-step process
  - F-test (Page 1-2; Table 1 – Pages 3-4)
  - t-test (Page 2, 5-6; Table 2 – Page 7)
- F-test
  - Compares variances
  - Tells you which t-test to perform
- t-test
  - Compares means
  - There are two types based on if the F-test passes or fails
  - Decisions are made based on whether the t-test passes or fails

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Practice Problems

Statistics

Question 1

The contractor has the following air void data from lot 8. What is the mean, standard deviation, and variance?

Contractor

Sublot	Air Voids (%)
8A	5.06
8B	4.73
8C	4.19
8D	3.64
8E	2.75

Question 1

The contractor has the following air void data from lot 8. What is the mean, standard deviation, and variance?

Contractor

Sublot	Air Voids (%)
8A	5.06
8B	4.73
8C	4.19
8D	3.64
8E	2.75

$n = 5$

$\bar{X} = 4.07\%$

$S = 0.92\%$

$S^2 = 0.85\%$

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Question 2

The contractor has the following air void data from lot 8. The Upper Spec Limit (USL) is 5.25% and the Lower Spec Limit (LSL) is 2.75%. What is the Lower Quality Index, Upper Quality Index, and the Percent Within Limits?

Contractor

Sublot	Air Voids (%)
8A	5.06
8B	4.73
8C	4.19
8D	3.64
8E	2.75

n = 5

$\bar{X} = 4.07\%$

$S = 0.92\%$

$S^2 = 0.85\%$

LSL = 2.75%

USL = 5.25%

$$Q_u = \frac{(USL - \bar{X})}{s}$$

$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$PWL = (PWL_U + PWL_L) - 100$

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Question 2

The contractor has the following air void data from lot 8. The Upper Spec Limit (USL) is 5.25% and the Lower Spec Limit (LSL) is 2.75%. What is the Lower Quality Index, Upper Quality Index, and the Percent Within Limits?

Contractor

Sublot	Air Voids (%)
8A	5.06
8B	4.73
8C	4.19
8D	3.64
8E	2.75

n = 5

$\bar{X}$ = 4.07%

S = 0.92%

S^2 = 0.85%

LSL = 2.75%

USL = 5.25%

$$Q_u = \frac{(5.25 - 4.07)}{0.92}$$

$$Q_u = \frac{(1.18)}{0.92} = 1.28$$

$$Q_L = \frac{(4.07 - 2.75)}{0.92}$$

$$Q_L = \frac{(1.32)}{0.92} = 1.43$$

$n = 5$

$Q_U = 1.28$

$PWL_U = 91.29$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.18	100.00	89.33	88.71	88.53	88.44	88.39	88.35	88.32	88.25	88.21	88.18	88.15	88.12
1.19	100.00	89.67	88.98	88.77	88.67	88.61	88.57	88.54	88.46	88.42	88.38	88.35	88.32
1.20	100.00	90.00	89.24	89.01	88.90	88.83	88.79	88.76	88.66	88.62	88.58	88.54	88.52
1.21	100.00	90.33	89.50	89.25	89.13	89.06	89.00	88.97	88.87	88.82	88.78	88.74	88.71
1.22	100.00	90.67	89.77	89.49	89.35	89.27	89.22	89.18	89.07	89.02	88.97	88.93	88.91
1.23	100.00	91.00	90.03	89.72	89.58	89.49	89.43	89.39	89.27	89.22	89.16	89.12	89.09
1.24	100.00	91.33	90.28	89.96	89.80	89.70	89.64	89.59	89.47	89.41	89.36	89.31	89.28
1.25	100.00	91.67	90.54	90.19	90.02	89.91	89.85	89.79	89.66	89.60	89.54	89.50	89.47
1.26	100.00	92.00	90.79	90.42	90.23	90.12	90.05	90.00	89.85	89.79	89.73	89.68	89.65
1.27	100.00	92.33	91.04	90.64	90.45	90.33	90.25	90.19	90.04	89.98	89.91	89.87	89.83
1.28	100.00	92.67	91.29	90.87	90.66	90.53	90.45	90.39	90.23	90.16	90.10	90.05	90.01
1.29	100.00	93.00	91.54	91.09	90.87	90.74	90.65	90.58	90.42	90.34	90.28	90.22	90.18
1.30	100.00	93.33	91.79	91.31	91.07	90.94	90.84	90.78	90.60	90.52	90.45	90.40	90.36
1.31	100.00	93.67	92.03	91.52	91.28	91.13	91.04	90.97	90.78	90.70	90.63	90.57	90.53
1.32	100.00	94.00	92.27	91.74	91.48	91.33	91.23	91.15	90.96	90.88	90.80	90.74	90.70
1.33	100.00	94.33	92.51	91.95	91.68	91.52	91.41	91.34	91.14	91.05	90.97	90.91	90.87
1.34	100.00	94.67	92.75	92.16	91.88	91.71	91.60	91.52	91.31	91.22	91.14	91.08	91.03

$n = 5$

$Q_L = 1.43$

$PWL_L = 94.77$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes											
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.35	100.00	95.00	92.98	92.37	92.08	91.90	91.78	91.70	91.48	91.39	91.31	91.24	91.19
1.36	100.00	95.33	93.21	92.58	92.27	92.09	91.96	91.88	91.65	91.56	91.47	91.40	91.35
1.37	100.00	95.67	93.44	92.78	92.46	92.27	92.14	92.05	91.82	91.72	91.63	91.56	91.51
1.38	100.00	96.00	93.67	92.98	92.65	92.45	92.32	92.23	91.99	91.88	91.79	91.72	91.67
1.39	100.00	96.33	93.90	93.18	92.83	92.63	92.49	92.40	92.15	92.04	91.95	91.88	91.82
1.40	100.00	96.67	94.12	93.37	93.02	92.81	92.67	92.56	92.31	92.20	92.10	92.03	91.98
1.41	100.00	97.00	94.34	93.57	93.20	92.98	92.83	92.73	92.47	92.36	92.26	92.18	92.13
1.42	100.00	97.33	94.56	93.76	93.38	93.15	93.00	92.90	92.63	92.51	92.41	92.33	92.27
1.43	100.00	97.67	94.77	93.95	93.55	93.32	93.17	93.06	92.78	92.66	92.56	92.48	92.42
1.44	100.00	98.00	94.98	94.13	93.73	93.49	93.33	93.22	92.93	92.81	92.70	92.62	92.56
1.45	100.00	98.33	95.19	94.32	93.90	93.65	93.49	93.37	93.08	92.96	92.85	92.76	92.70
1.46	100.00	98.67	95.40	94.50	94.07	93.81	93.65	93.53	93.23	93.10	92.99	92.90	92.84
1.47	100.00	99.00	95.61	94.67	94.23	93.97	93.80	93.68	93.37	93.25	93.13	93.04	92.98
1.48	100.00	99.33	95.81	94.85	94.40	94.13	93.96	93.83	93.52	93.39	93.27	93.18	93.12
1.49	100.00	99.67	96.01	95.02	94.56	94.29	94.11	93.98	93.66	93.52	93.40	93.31	93.25
1.50	100.00	100.00	96.20	95.19	94.72	94.44	94.26	94.13	93.80	93.66	93.54	93.45	93.38
1.51	100.00	100.00	96.39	95.36	94.87	94.59	94.40	94.27	93.94	93.80	93.67	93.58	93.51
1.52	100.00	100.00	96.58	95.53	95.03	94.74	94.55	94.41	94.07	93.93	93.80	93.71	93.64
1.53	100.00	100.00	96.77	95.69	95.18	94.88	94.69	94.55	94.20	94.06	93.93	93.83	93.76

Question 2

The contractor has the following air void data from lot 8. The Upper Spec Limit (USL) is 5.25% and the Lower Spec Limit (LSL) is 2.75%. What is the Lower Quality Index, Upper Quality Index, and the Percent Within Limits?

Contractor

Sublot	Air Voids (%)
8A	5.06
8B	4.73
8C	4.19
8D	3.64
8E	2.75

n = 5

$\bar{X}$ = 4.07%

S = 0.92%

S^2 = 0.85%

LSL = 2.75%

USL = 5.25%

$Q_u = 1.28$

$Q_L = 1.43$

$PWL = (PWL_U + PWL_L) - 100$

$PWL = (91.29 + 94.77) - 100$

$PWL = (186.06) - 100$

$PWL = 86.06$

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Question 3

The contractor has the following air void data from lot 8. What is the 4 point moving average for test #4 and test #5?

Contractor

	Sublot	Air Voids (%)
x_1:	8A	5.06
x_2:	8B	4.73
x_3:	8C	4.19
x_4:	8D	3.64
x_5:	8E	2.75

$$x_{ma_i} = \frac{xi + x(i-1) + x(i-2) + x(i-3)}{4}$$

$$x_{ma_4} = \frac{x_4 + x_3 + x_2 + x_1}{4}$$

$$x_{ma_4} = \frac{3.64 + 4.19 + 4.73 + 5.06}{4}$$

$$x_{ma_4} = \frac{17.62}{4} = 4.40\%$$

Question 3

The contractor has the following air void data from lot 8. What is the 4 point moving average for test #4 and test #5?

Contractor

	Sublot	Air Voids (%)
x_1:	8A	5.06
x_2:	8B	4.73
x_3:	8C	4.19
x_4:	8D	3.64
x_5:	8E	2.75

$$x_{ma_i} = \frac{xi + x(i-1) + x(i-2) + x(i-3)}{4}$$

$$x_{ma_5} = \frac{x_5 + x_4 + x_3 + x_2}{4}$$

$$x_{ma_5} = \frac{2.75 + 3.64 + 4.19 + 4.73}{4}$$

$$x_{ma_5} = \frac{15.31}{4} = 3.83\%$$

Question 4

The following in-place density data has been calculated from two different sets of test results from lot 9. The first set is from the contractor quality control tests and the second set of data is from KDOT verification tests. The Lower Specification Limit is 91.00%. Use a level of significance (a) = 1%.

Contractor

Sublot	Density ($\%G_{mm}$)
9A1	92.10
9A2	93.33
9B1	90.72
9B2	91.15
9C1	92.27
9C2	92.23
9D1	89.51
9D2	91.15
9E1	91.84
9E2	91.27

KDOT

Sublot	Density ($\%G_{mm}$)
9A	91.84
9B	92.66
9C	91.87
9D	89.68
9E	91.49

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Question 4

From the in-place density data from lot 9, Determine the following:

1. The Basic Statistics for both sets of data.
2. Determine if the t-test passes or fails.
3. If the t-test passes, use the Contractor's data to determine the Percent Within Limits.
4. If the t-test fails, use KDOT's data to determine the Percent Within Limits

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Question 4

From the in-place density data from lot 9, Determine the following:

1. The Basic Statistics for both sets of data.

Contractor

Sublot	Density ($\%G_{mm}$)
9A1	92.10
9A2	93.33
9B1	90.72
9B2	91.15
9C1	92.27
9C2	92.23
9D1	89.51
9D2	91.15
9E1	91.84
9E2	91.27

$n_c = 10$

$\bar{x}_c = 91.56$

$s_c = 1.04$

$s_c{}^2 = 1.08$

KDOT

Sublot	Density ($\%G_{mm}$)
9A	91.84
9B	92.66
9C	91.87
9D	89.68
9E	91.49

$n_v = 5$

$\bar{x}_v = 91.51$

$s_v = 1.11$

$s_v{}^2 = 1.23$

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Question 4

From the in-place density data from lot 9, Determine the following:

2. Determine if the t-test passes or fails.

Compute F-statistic value: $F = \frac{s_c^2}{s_v^2}$ or $\frac{s_v^2}{s_c^2}$ $\quad F = \frac{1.23}{1.08} = 1.14$

Determine F_{CRIT}

Determine Degrees of Freedom

df$_{num}$: $n_v = 5$, thus df$_{num}$ = 5 – 1 = 4

df$_{den}$: $n_c = 10$, thus df$_{den}$ = 10 – 1 = 9

From Table 1 in 5.2.6: $F_{CRIT} = 7.96$

Since F (1.14) < F_{CRIT} (7.96), use the pooled estimate for variance and pooled degrees of freedom when calculating the t-statistic

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Question 4

From the in-place density data from lot 9, Determine the following:

2. Determine if the t-test passes or fails.

   Compute t-statistic value:

   Calculate the pooled variance: $s_p^2 = \frac{s_c^2(n_c - 1) + s_v^2(n_v - 1)}{n_c + n_v - 2}$

$$s_p{}^2 = \frac{1.08(10 - 1) + 1.23\,(5 - 1)}{10 + 5\ - 2} = \frac{9.72\ + 4.92}{13} \longrightarrow \boldsymbol{s_p{}^2} = \mathbf{1.126}$$

Compute t-statistic value

$$t = \frac{|\overline{X}_c - \overline{X}_v|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_v}}} = \frac{|\ 91.56\ \text{-}\ 91.51|}{\sqrt{\frac{1.126}{10} + \frac{1.126}{5}}} = \frac{|\ 0.05\ |}{\sqrt{0.1126 + 0.225}} = \frac{0.05}{0.581} \longrightarrow \boldsymbol{t} = \mathbf{0.086}$$

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Question 4

From the in-place density data from lot 9, Determine the following:

2. Determine if the t-test passes or fails.

   Determine t_{CRIT}:

   $t = 0.086$

   Pooled Degrees of Freedom = $n_c + n_v - 2$ = 10 + 5 – 2 = 13

   Level of Significance (a) = 1% (Given)

   From Table 2 in 5.2.6: t_{CRIT} = 3.012

   Since t (0.086) < t_{CRIT} (3.012), The t-test passes, the sample means are assumed equal and the contractor’s test results are used for the pay adjustments.

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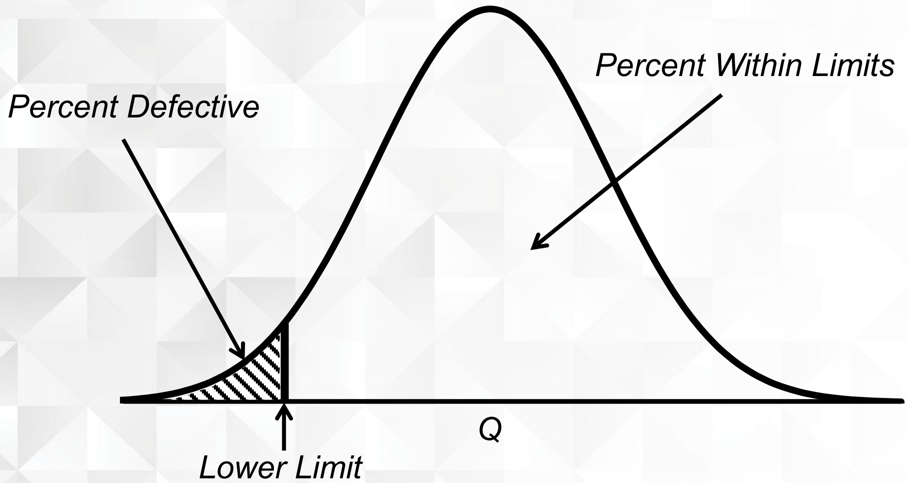
Quality Level Analysis
Concept of Percent Defective

Single Limit Specification



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From the in-place density data from lot 9, Determine the following:

3. If the t-test passes, use the Contractor's data to determine the Percent Within Limits.

Using the contractor's data we have: $\bar{x}$ = 91.56% s = 1.04% n = 10 LSL = 91.00% (Given)

$$Q_L = \frac{(\bar{X} - LSL)}{s}$$

$$Q_L = \frac{(91.56 - 91.00)}{1.04} = \frac{(0.56)}{1.04} \longrightarrow Q_L = 0.54$$

Enter Table 2 in 5.2.1 with $Q_L = 0.54$ and $n = 10$ ⟶ $PWL_L = 70.02\%$

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Question 4

From the in-place density data from lot 9, Determine the following:

1. The Basic Statistics for both sets of data.
2. Determine if the t-test passes or fails.
3. If the t-test passes, use the Contractor’s data to determine the Percent Within Limits.
4. If the t-test fails, use KDOT’s data to determine the Percent Within Limits

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