

Basic Math Test Study Guide

The Basic Math Test covers basic skills need by a Technician to succeed in passing CIT² courses and in understanding and performing KDOT test procedures. This guide covers most of the basic principles needed. It is not intended to be self study or all inclusive. The concepts should be presented by a knowledgeable person.

Things to know:

Calculator operation

- Store/recall

- Pi

- Square/square root

- Parenthesis

Order of operations

- PEMDAS, “**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally”

- Fraction Lines

Variables

Fractions/Decimals

Percentages

Rounding (KDOT method)

Area

- Square/rectangle

- Circles

 - Radius

 - Diameter

- Triangles

- Surface area of a cube

Volume

- Rectangular base

- Circular Base

Solving equations

Word problems

Order of operations

PEMDAS, “Please Excuse My Dear Aunt Sally”

Mnemonic to remember:

Going from left to right in an equation, operations inside

Parenthesis, such as $(x+y)$, are done first

Exponents are done next. $X*Y^2$

Multiplication; AB , AxB , $A*B$ or

Division operations; A/B or $A\div B$ are done next

Addition, $2+X$ and

Subtraction $X-\alpha$ are done last

Fraction Lines

Operations or groups of operations above and below the fraction line are completed separately and then the result above the line is divided by the result below the line.

$$\frac{(2+A)A}{2AC} =$$

Variables

Variable: A letter or symbol used to represent a value that may change.

Constant: A value that does not change which may be represented by a symbol. Pi for example has a value of approximately 3.14 and is represented by the symbol π .

1.

Solve the following for $X=2$

$$X*2=$$

$$X/2=$$

$$X+5=$$

$$3X=$$

$$X^2=$$

Solve the above if $X=9$

.....

Variables

2.

Solve for X

$$3*9=X$$

$$2*5+1=X$$

$$3X=9$$

3.

Solve these problems if A=5 and C=3

$$A+2C=$$

$$(A*C)/(C*A) =$$

$$3A + C*A=$$

$$(3A+7C)/C=$$

Solve for G

$$G = \frac{A}{A-C}$$

$$\frac{(2+A)*(C^2+A)}{(2AC)} =$$

4.

$$\alpha=1.253, z=2.056, A=5, Y_a=10$$

Solve:

$$\alpha+z+Y_a-A=$$

$$\alpha\left(\frac{z*Y_a}{A}\right)=$$

$$(\alpha*\pi)/Y_a=$$

Answers

1. For x=2, 4, 1, 7, 6, 4
For x=9, 18, 4.5, 14, 27, 81

2. 27, 11, 3

3. 11, 1, 30, 12, 2.5, 3.2667

4. 8.309, 5.152336, approx. 0.39

Fractions to Decimal

To convert a fraction to decimal, divide the number above the line (numerator) by the number below the line (denominator).

$$\frac{1}{3} =$$

$$\frac{9}{27} =$$

$$\frac{9}{5} =$$

$$\frac{1.5}{0.25} =$$

$$\frac{1.25}{100} =$$

0.3333, 0.3333, 1.8, 6, 0.0125

Percentages

Fractions can be changed to percentages by dividing the numerator by the denominator, the same as changing to a decimal as shown above, and then multiplying the result by 100.

For example $\frac{3}{8}$ converted to a decimal is 3 divided by 8 or 0.375. Multiplying that result by 100 gives us 37.5. 3 is 37.5% of 8.

1. Convert the following to percentages:

$$\frac{1}{4}$$

$$\frac{1}{3}$$

$$\frac{1}{2}$$

$$\frac{2}{3}$$

Dividing a part of a group by the whole works the same way giving us a decimal that is then multiplied by 100 to give us a percentage in the same fashion as above.

2. Out of 100 students, 27 voted to have orange juice for lunch. What percentage of students wanted orange juice?

3. 13 parts out of 1500 parts manufactured were found to be defective. What percentage of parts was defective? What percentage of parts is not defective?

Percentages

Answers:

1.

25%

33.33%

50%

66.67%

2. 27%

3. 0.87%, 99.13%

Rounding, KDOT method (from 5.17.05)

(1) Determine the last significant digit to be retained.

(2) If the figure to the right of the last place to be retained is less than 5, retain the last place as it is.

(3) If the figure to the right of the last place to be retained is greater than 5, increase the last place by 1.

(4) If the figure to the right of the last place to be retained is exactly 5, retain the last place as it is if it is an even number or increase the last place by 1, if it is an odd number.

(5) Examples: The following table indicates correct rounding-off procedures.

Observed or Calculated Value	Rounded to	Rounded Off Value	Specified Limit	Conform with a Specified Value
56.4	1 %	56		no
56.5		56	57% min.	no
56.6		57		yes
0.54	0.1 %	0.5		yes
0.55		0.6	0.5% max.	no
0.56		0.6		no

The rounded-off value should be obtained in one step by direct rounding off of the most precise value available and not in two or more successive roundings. For example, 89,490 rounded off to the nearest 1000 is at once 89,000; it would be incorrect to round off first to the nearest 100, giving 89,500 and then to the nearest 1000 giving 90,000.

Note that some test procedures require a different rounding procedure. KT-55 requires a value be calculated to 0.1 and rounded to the next higher whole number if the calculated value was not a whole number.

1. Round to the whole number using standard KDOT rounding.

1.875

1.5

23.499987

25.01

2. Round the values stated in 1. using the method stated for KT-55 above

1. 2, 2, 23, 25

2. 2, 2, 24, 25

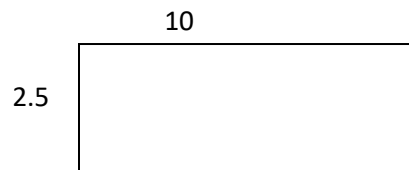
Surface area

Area of a Rectangle

The surface area of a square or rectangle is determined by multiplying the length by the width. In the figure below we multiply 2.5 by 10 to get 25. Area is expressed in square units. This could be square inches (in²), square feet (ft²), square miles or whatever units are being used to measure with. The units must be the same or converted to the same units.

If the example below was 2.5 inches tall and 10 feet long we would convert the 10 feet to inches by multiplying 10 (feet) by 12 (inches) to get 120 inches and then multiply that by 2.5 to get 300 square inches.

If the 2.5 was inches and the 10 was feet, we could change the 2.5 inches to feet by dividing 2.5 by 12 for a result of approximately 0.2083 (feet) and then multiply by 10 (feet) getting an answer of 2.083 square feet (ft²).



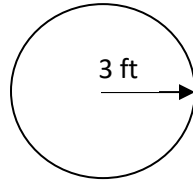
Area of a Circle

The area of a circle is determined by multiplying the value of Pi (π) by the square of radius. Written as $\text{area} = \pi r^2$. Pi is approximately equal to 3.1416 and the value can be found on most scientific calculators.

Surface area

Area of a Circle

The radius is the distance from the center point of a circle to any point on the circle or one half the diameter. In the figure below the radius is 3 feet so the area would be equal $\pi * 3^2$ or approximately 28.27 ft².

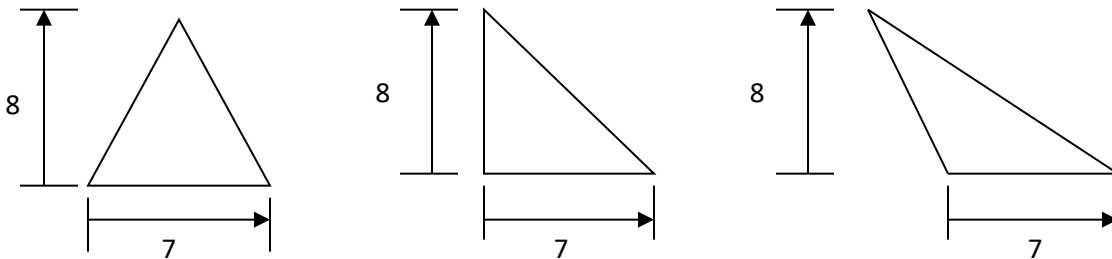


The diameter of a circle is the length of a line through the center point and touching the edge of the circle on either side of the center point. In the example above the diameter would be 6 feet. If the diameter was given, as in the problem: “The diameter of a circle is 10 feet. What is the area of the circle?” The diameter is divided by 2, that answer is squared and then multiplied by π . As in the equation: $\left(\frac{10}{2}\right)^2 * \pi$ or 78.54 ft².

Area of a Triangle

The area of a triangle is determined by multiplying $\frac{1}{2}$ times the length of the base times the height. $\frac{1}{2} * b * h$. Note the height is perpendicular to the base and may or may not equal the length of a side.

The three triangles below all have the same area. $\frac{1}{2} * 7 * 8$ or 28.



Surface area of a cube

All three dimensional objects such as a sphere, cube or a particle of sand have a property known as surface area. The surface of a cube is composed of 6 sides that rectangular in shape. The surface area of a cube is the sum of the area of all of the sides. The area of each side is the length times the width as in the example at the beginning of this section. In a cube the length

and width and height are the same. So, a four foot cube would have an area on one side of 4×4 or 16 ft^2 , the 16 ft^2 times 6 sides would equal 96 ft^2 . A four foot wide cube has a surface area of 96 ft^2 .

1. What is area of a 5 foot square?
2. What is area of a rectangle that is 14 inches long by 9 inches high?
3. What is area circle having a radius of 25 meters?
4. What is area circle having a diameter of 25 meters?
5. What is area of a triangle that has a base 24 inches long and is 18 inches high?
6. What is area of a 5 foot square *in square inches*?
7. What is area of a rectangle that is 14 inches long by 9 inches high *in square feet*?
8. What is the surface area of an 8 inch cube?

Answers

1. 25 ft^2 2. 126 in^2 3. 1963.5 m^2 4. 490.87 m^2 5. 216 in^2 6. 3600 in^2 7. 0.875 ft^2 8. 384 in^2

Volume

The volume of an object with an equal base and top that are perpendicular to the sides such as a cylinder or a box is equal to the area of the base times the height. Volume is expressed in cubic units such as cubic feet (ft^3).

Calculate the area of the rectangular or circular base as shown in the section on area then multiply by the height of the object to determine the volume.

What is the volume of the two figures shown?

Fig. 1

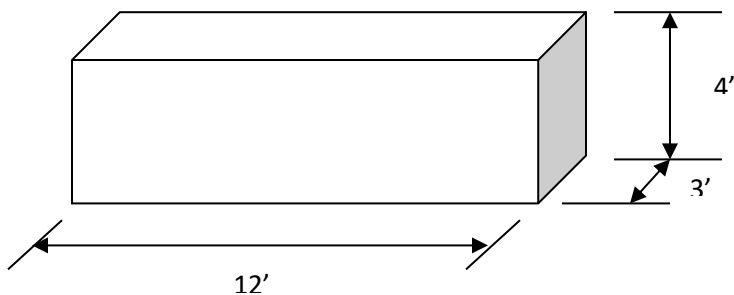
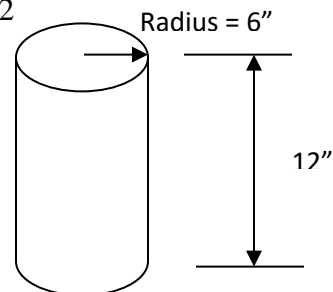


Fig. 2



Volume

Answers: Fig.1 144 ft³, Fig. 2 1357.17 in³

Calculate the volume of a cube measuring 6 inches on a side.

Answer: 216 in³

Calculate the volume of a cylinder having a diameter of 3 feet and a height of 20 feet.

Answer: 141.37 ft³

Solving equations

Remembering that what is done to one side of an equation must also be done to the other side can help solve some math problem.

The equation $3x=12$, is solved by dividing both sides by 3. $3x$ divided by 3 equals x and 12 divided by 3 equals 4. Therefore, $x=4$

$$\begin{aligned}3x &= 12 \\ \frac{3}{3}x &= \frac{12}{3} \\ 1x &= 4 \\ x &= 4\end{aligned}$$

Similarly we can multiply both sides of an equation.

$$\begin{aligned}\frac{1}{4}z &= 2 \\ 4 \cdot \frac{1}{4}z &= 2 \cdot 4 \\ \frac{4}{4}z &= 8 \\ 1z &= 8 \\ z &= 8\end{aligned}$$

We can add or subtract if we do it to both sides of the equation.

In the equation $3x+3=15$ we can eliminate the 3 on the left side by subtracting 3 from both sides

$$\begin{aligned}3x+3 &= 15 \\ 3x+3-3 &= 15-3 \\ 3x+0 &= 12 \\ 3x &= 12, \text{ then solving as above by dividing each side by 3, } x=4\end{aligned}$$

We can square or take the square root of both sides of an equation.

$$36=a^2$$

$$\sqrt{36}=\sqrt{a^2}$$

$$6=\sqrt{a^2}$$

The square root of a^2 equals a

$$6=a$$

$$\sqrt{y}=9$$

$$(\sqrt{y})^2=9^2$$

$$y=81$$

The square of a square root equals the value itself

$$y=81$$

Word problems

We have already seen word problems in the examples above, like “What is area of a rectangle that is 14 inches long by 9 inches high?” When a problem is given in words, it should be written into a mathematical form that can be solved, an equation. To solve the given problem we know that we are trying to find the area. Area is length times width, written as $\text{area} = L \times W$. The length given is 14” and the width is 9” so, $\text{area} = 14 \times 9$. $14 \times 9 = 126$ so the area = 14”x 9” and area = 126 in².

We were given the problem “Calculate the volume of a cylinder having a radius of 3 feet and a height of 20 feet” in the Volume section above. We know the volume of a cylinder equals $\pi r^2 \times \text{height}$. Writing out the problem mathematically we get

$$\text{Volume} = \pi r^2 \times H$$

Where r = the radius and H = height and π = the constant Pi, approximately 3.1416.

Plugging in the given value for r we get

$$\text{Volume} = \pi 3^2 \times H$$

Plugging in the given value for H we get

$$\text{Volume} = \pi 3^2 \times 20$$

Plugging in the given value for Pi we get

$$\text{Volume} = 3.1416 \times 3^2 \times 20$$

$3^2 = 9$, so we have

Volume = $3.1416 \times 9 \text{ft}^2 \times 20 \text{ft}$

Then multiplying $3.1416 \times 9 \times 20$ we get approximately 565.49 and since volume is expressed in cubic feet ($\text{ft}^2 \times \text{ft} = \text{ft}^3$) we have:

Volume = 565.49 ft^3

When given a word problem, write out the formula necessary to provide the answer then plug in the given values and solve mathematically.