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Statistics Handbook

Certified Inspector Training Program

Kansas
Department of Transportation
AD ASTRA PER ASPERA

Statistics Handbook

Table of Contents

Click on the section name below to be taken to the correct page.

5.2. QUALITY CONTROL /QUALITY ASSURANCE

5.2.1. STATISTICS

1. ACKNOWLEDGEMENT

This paper has been copied directly from the HMA Manual with a few modifications from the original version. The original version was prepared by Dr. Mustaque Hossain. Ph. D., P.E., Kansas State University.

2. BACKGROUND

2.1 American industries have defined the Quality Control/Quality Assurance (QC/QA) concept to fit within their particular application and there is no doubt that the "working" definition differs from industry to industry. In the highway community those Contractors, suppliers, and Public Agencies (Agency) that have implemented a QC/QA program probably have their own definition as well. It is important that a concise and logical definition of QC/QA be adopted and be supported by all members within a single industry. For the highway community the QC/QA concept must be defined so that Contractors, suppliers and Agencies can identify with a basic concept and proceed to establish their respective programs. The American Association of State Highway and Transportation Officials (AASHTO) has defined QC/QA in AASHTO R-10-06 (2011) in the following manner:

2.2. Quality Control: The system used by a contractor to monitor, assess and adjust their production or placement processes to ensure that the final product will meet the specified level of quality. Quality control includes sampling, testing, inspection and corrective action (where required) to maintain continuous control of a production or placement process.

2.3. Quality Assurance: All those planned and systematic actions necessary to provide confidence that a product or facility will perform satisfactorily in service; or (2) making sure the quality of a product is what it should be.

3. STANDARD DEFINITIONS FOR QC/QA APPLICATIONS

The following terms are defined in the AASHTO R-10-06 (2011) *except Agency*. All other shall be consistent with the **2008 AASHTO Guide Specifications For Highway Construction**.

3.1. Agency: The State Highway or Transportation Department, Commission, or other organization, constituted under State or Commonwealth laws, that administers highway or transportation work.

The term Agency was chosen for the purpose of consistency, as this document is intended for use by any governing organization attempting to prepare specifications for the purpose of highway or transportation work.

3.2. Acceptable Quality Level (**AQL**): The level of established actual quality for a quality characteristic that is fully acceptable.

3.3. Buyer: See *Agency* above.

3.4. Buyer's Risk: Also called *agency's risk*, or *risk of a Type II* or *beta (β) error*. It is the risk to the agency of accepting rejectable quality level (*RQL*) material or workmanship.

3.5. Certified Technician: A technician certified by some agency as proficient in performing certain duties.

3.6. Disincentive: A pre-established decrease in payment to the contractor applied to a contract bid item for which the level of materials quality and workmanship, determined by statistical means, does not meet the specified acceptable quality level *(AQL)*. The disincentive is usually expressed as a percentage of the original Contract bid-price.

3.7. Incentive/disincentive provision (for quality): A pay adjustment schedule which functions to motivate the contractor to provide a high level of quality.

3.8. Lower Specification Limit (**LSL**): The lower statistically based limiting value associated with a quality characteristic and used to evaluate the acceptability of a lot.

3.9. Percent Within Limits (PWL): The percentage of the lot falling above a lower specification limit, beneath an upper specification limit, or between upper and lower specification limits.

3.10. Quality Assurance: All those planned and systematic actions necessary to provide confidence that a product or facility will perform satisfactorily in service; or (2) making sure the quality of a product is what it should be.

3.11. Quality Control: Also called *process control*. The system used by a contractor to monitor, assess and adjust their production or placement processes to ensure that the final product will meet the specified level of quality. Quality control includes sampling, testing, inspection and corrective action (where required) to maintain continuous control of a production or placement process.

3.12. Quality Control Plan: A project-specific document prepared by the contractor that identifies all QC personnel and procedures that will be used to maintain all production and placement processes "in control" and meet the agency specification requirements. The document also addresses actions to be taken in the event that a process goes "out of control".

3.13. Quality Level Analysis: A statistical procedure that provides an estimate of the percentage of a given lot that is within specification limits (*PWL*) or outside specifications limits (*PD*).

3.14. Rejectable Quality Level (RQL): The level of established actual quality for a quality characteristic that is rejectable when using a particular quality measure.

3.15. Seller's Risk (α): Also called *contractor's risk,* or *risk of a type 1* or *alpha (α) error.* The risk to the contractor of having acceptable quality level *(AQL)* material or workmanship rejected.

3.16. Target Value: The value that is placed on a quality characteristic that represents the mean of the expected distribution of the specified population.

3.17. Upper Specification Limit (USL): The upper statistically based limiting value associated with a quality characteristic and used with a quality measure to evaluate the quality of a lot.

4. MISCELLANEOUS DEFINITIONS RELATED TO QC/QA

4.1. Lot: An isolated quantity of material which is produced from a single source under similar conditions. A lot is a measured amount of construction assumed to be produced by the same process. For example, the placement of 4,000 tons of hot-mix asphalt (HMA) or one days production for concrete paving. Also referred to as population in statistical analysis.

4.2. Sublot: Sublots are equal divisions (i.e. portions) of a lot. A lot is divided into sublots for sampling purposes. For example, if a lot is considered to be 4,000 tons of HMA and the specification requires that the lot be divided into four sublots, the size of each sublot would be 1,000 tons (4,000 /4). For concrete paving, a lot is considered to be one days production and the specification requires that the lot be subdivided into five sublots as shown in **Figure 1.**

4.3. Sample: Each individual quantity of material collected for test. A portion of lot.
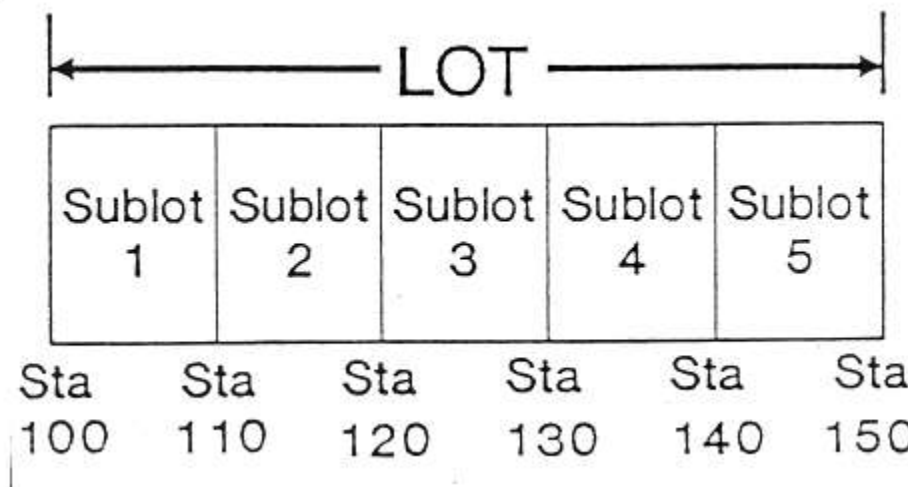
Figure 1. Lot and sublots in a highway construction setting

5. INTRODUCTORY STATISTICAL TERMS

5.1. Average or Mean $(\bar{x})$: Arithmetic mean or average determined for a number of variables (x_i) as below:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x_i}{n} \tag{1}$$

5.1.1. *Example: Find the arithmetic mean or average for the asphalt content of six Superpave mix sublots given as: 5.4, 5.8, 6.2, 5.4, 5.4 and 6.0%.*

$$\bar{x} = \frac{5.4+5.8+6.2+5.4+5.4+6.0}{6} = \frac{34.2}{6} = 5.7$$

5.1.2. *Example: Find the arithmetic mean or average for the percent air in the concrete mix of six sublots given as: 6.6, 6.2, 5.5, 7.8, 6.9 and 6.6%.*

$$\bar{x} = \frac{6.6+6.2+5.5+7.8+6.9+6.6}{6} = \frac{39.6}{6} = 6.6$$

5.2. Moving Average: Average computed based on a fixed set of continuous data points. For KDOT, the w represents the number of tests within a lot:

$$x_{ma} = \frac{x_{i-3} + x_{i-2} + x_{i-1} + x_i}{w} \tag{2}$$

5.2.1. *Example: Find the 4-point moving average for the above asphalt content data:*

Asphalt content (%)	4-point moving average
5.4	-
5.8	-
6.2	-
5.4	5.7
5.4	5.7
6.0	5.8

5.2.2. *Example: Find the 4-point moving average for the above percent air content data:*

Air content (%)	4-point moving average
6.6	-
6.2	-
5.5	-
7.8	6.5
6.9	6.6
6.6	6.7

5.3. Range (R): Range is the difference between the largest and smallest values. A simple measure of variability.

$$R = x_{max} - x_{min} \tag{3}$$

5.3.1. *Example: Find the Range (R) for the asphalt content data in 5.2.1. above:*

$$R = 6.2 - 5.4 = 0.8\%$$

5.3.2. *Example: Find the Range (R) for the air content data in 5.2.2. above:*

$$R = 7.8 - 5.5 = 2.3\%$$

5.4. Sample Standard Deviation (s) : Standard deviation is the root mean square of the deviation from the mean. This is a better measure of variability than range and is computed as below:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \qquad (4)$$

where, n is the sample size.

5.4.1. *Example: Find the standard deviation for the asphalt content data given below:*

x_i	$\bar{x}$	x_i-$\bar{x}$	$(x_i$-$\bar{x})^2$
5.4	5.7	-0.3	0.09
5.8	5.7	0.1	0.01
6.2	5.7	0.5	0.25
5.4	5.7	-0.3	0.09
5.4	5.7	-0.3	0.09
6.0	5.7	0.3	0.09
n =6			$\Sigma = 0.62$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{0.62}{6-1}} = 0.35$$

5.4.2. *Example: Find the standard deviation for the air content data given below:*

x_i	$\bar{x}$	x_i-$\bar{x}$	$(x_i$-$\bar{x})^2$
6.6	6.6	0.0	0.00
6.2	6.6	-0.4	0.16
5.5	6.6	-1.1	1.21
7.8	6.6	1.2	1.44
6.9	6.6	0.3	0.09
6.6	6.6	0.0	0.00
n =6			$\Sigma = 2.90$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{2.90}{6-1}} = 0.76$$

5.5. Population Standard Deviation (σ): When the sample size n is large (usually greater than 30), the standard deviation obtained is for the population. The equation is same as in (4) <u>except that the denominator is replaced by n</u>.

5.6. Variance (s^2): Sample variance is simply the square of the sample standard deviation.

5.6.1. *Example: Find the variance of the asphalt content data given above:*

$s = 0.35;$ $s^2=(0.35)^2=0.123$

5.6.2. *Example: Find the variance of the air content data given above:*

$s = 0.76;$ $s^2=(0.76)^2=0.578$

5.7. Coefficient of Variation (COV): The coefficient of variation is defined as the standard deviation as a percentage of the mean. It is an additional measure of variability and is calculated as:

$$C.O.V.(\%) = \frac{s}{\bar{x}} \times 100 \qquad\qquad (5)$$

5.7.1. *Example: Find the coefficient of variation of the asphalt content data given above:*

$$C.O.V.(\%) = \frac{0.35}{5.7} \times 100 = 6.1\%$$

5.7.2. *Example: Find the coefficient of variation of the asphalt content data given above:*

$$C.O.V.(\%) = \frac{0.76}{6.6} \times 100 = 11.5\%$$

5.8. Normal Distribution Curve: It is a typical "bell-shaped" symmetrical curve which usually will describe the distribution of engineering measurements, e.g. test results of HMA or concrete mixes. **Figures 2, 3 and 4** show various examples of normal distribution curves and how they vary but are interrelated.
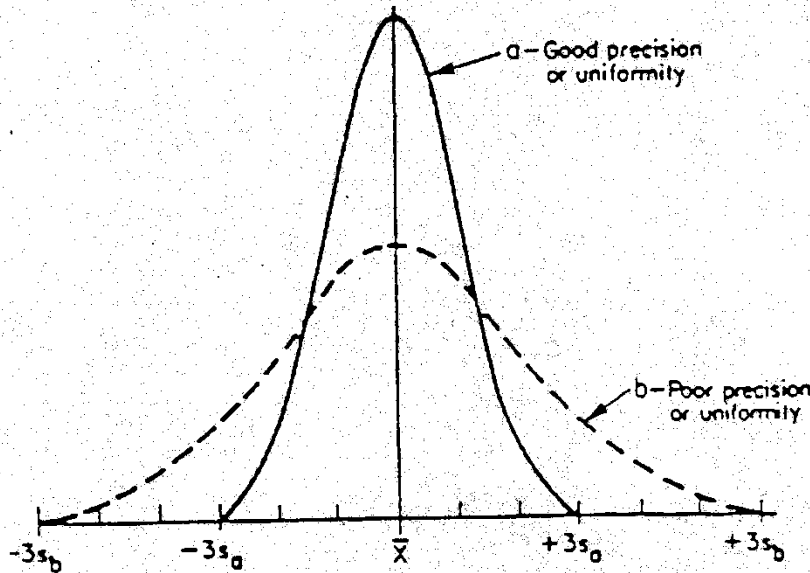
Figure 2. Normal distribution curves
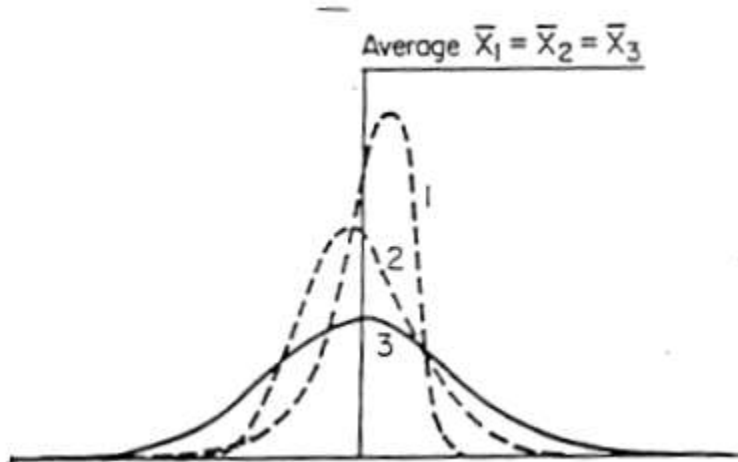
Figure 3. Quite different distributions may have the same average

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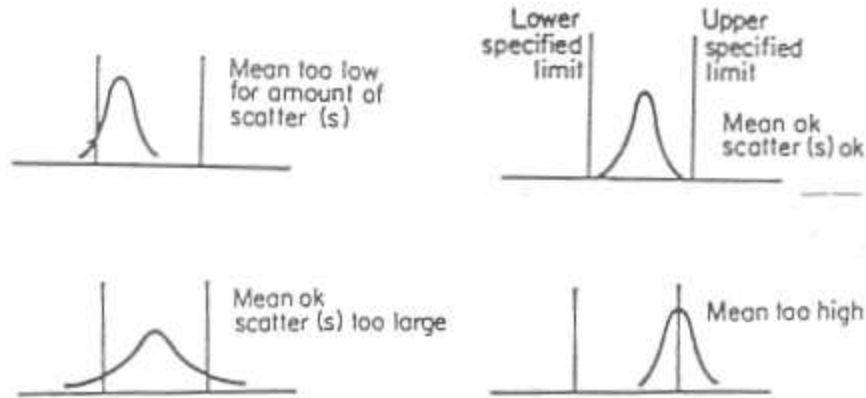
Figure 4. Process control related to specification limits

5.9. Control charts: Control charts are horizontal line charts. The horizontal lines (for single test results or for "average" type charts) generally consist of a central line at the specified average and an upper line at the specified upper acceptance limit and a lower line at the lower acceptance limit (if both are applicable) - for an "acceptance" control chart. **Figure 5** shows a typical control chart for average 9.5 mm aggregate size (percent retained). Control charts are very helpful for identifying possible problems. Examples are provided in **Figures 6, 7, 8, and 9.** It is to be noted that if these charts are plotted using individual test results, then the chance causes cannot be distinguished from assignable causes. However, the moving average tends to smooth out chance variations and a control chart based on moving averages can be used to indicate significant trends due to variation in materials and processes.
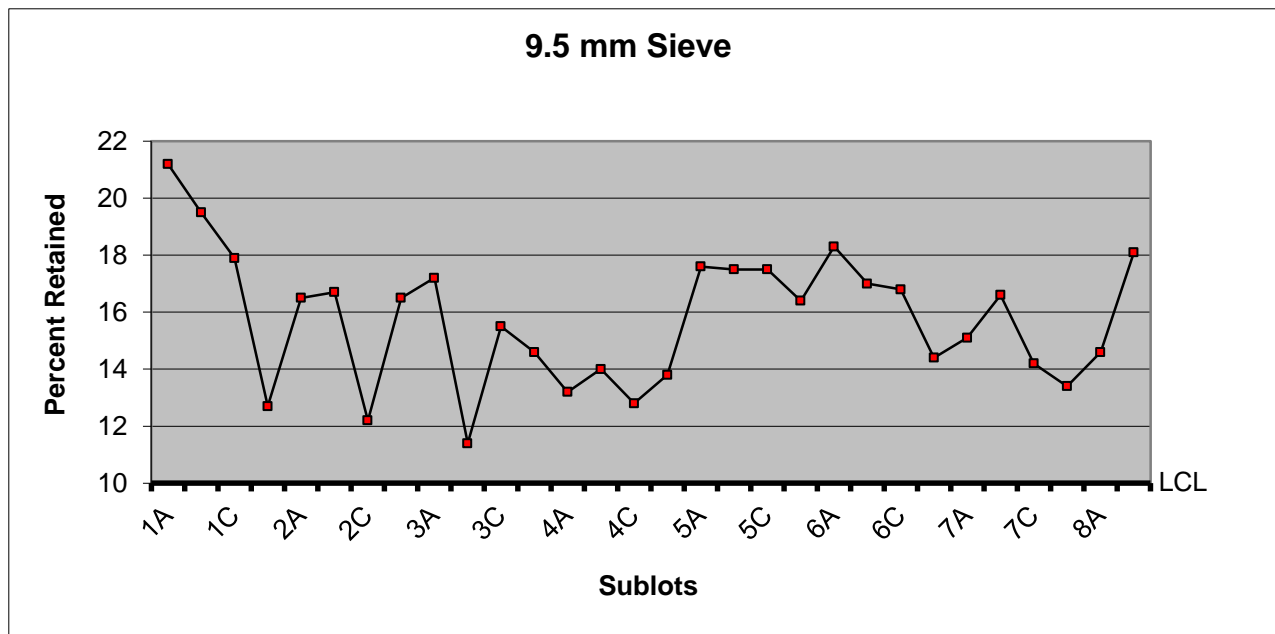
Figure 5. A typical control chart
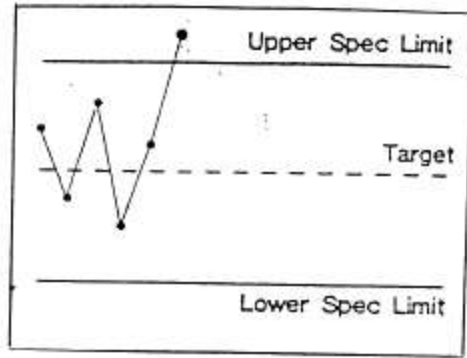
Figure 6. A point outside the upper specification limits
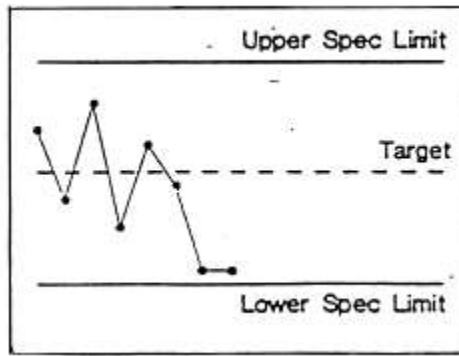
Figure 7. Two consecutive points near the upper or lower specification limits
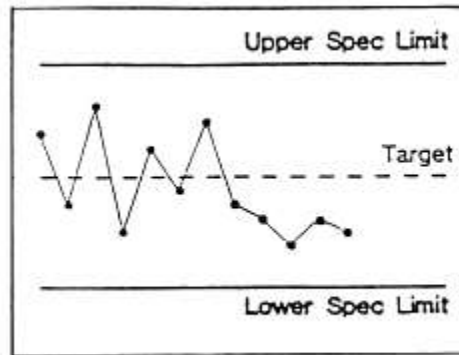
Figure 8. Five consecutive points on one side of the center target value
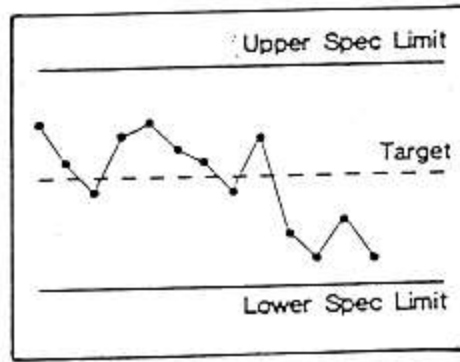
Figure 9. A sudden change in the level of results

5.10. Statistical Control charts: It is to be noted that regardless of the shape of the normal curve and spread (s), 68.26% of the test results will be within ± 1s, 95.44% within ± 2 s and 99.74% within ± 3 s. Thus, it is apparent that a control chart based on the test statistics (x and s) could be easily developed. The center line could be the mean of k sample means, each based n tests. The Upper Specification Limit (USL) and the Lower Specification Limit (LSL) can be fixed based on multiples of s resulting in a typical statistical control chart as shown in **Figure 10.**

5.11. Random Number: A number selected entirely by chance as from a table of random numbers as shown in **Table 1 of the KDOT Construction Manual Section 5.2.2.2**. (A scientific calculator can also generate random number; however, this method needs to be approved by the District Materials Engineer).
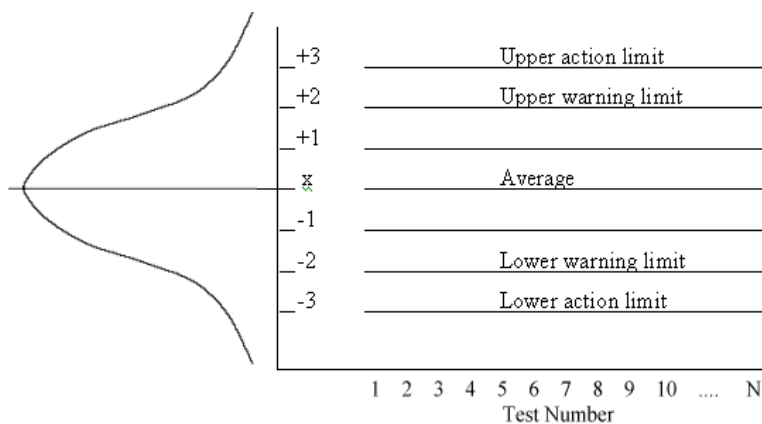
Figure 10. Statistical control chart

5.12. Moving Average

Within KDOT's QC/QA specifications, moving averages are to be plotted on the same chart as the individual test results. This is a simple process to include on the graphs. The number of tests in each average will be dictated by the specifications for the project. For the following illustration, a 4-point moving average will be considered.

Table 1: Individual Asphalt Content Test Data

Sublot	AC%	
1A	5.47	
1B	5.46	
1C	5.45	
1D	4.73	1st avg = (5.47 + 5.46 + 5.45 + 4.73)/4 = 5.28
2A	5.53	2nd avg = (5.46 + 5.45 + 4.73 + 5.53)/4 = 5.29
2B	5.41	3rd avg = (5.45 + 4.73 + 5.53 + 5.41)/4 = 5.28
2C	5.55	4th avg = (4.73 + 5.53 + 5.41 + 5.55)/4 = 5.31
2D	5.73	5th avg = (5.53 + 5.41 + 5.55 + 5.73)/4 = 5.56
3A	5.68	6th avg = (5.41 + 5.55 + 5.73 + 5.68)/4 = 5.59
3B	6.01	7th avg = (5.55 + 5.73 + 5.68 + 6.01)/4 = 5.74
3C	5.79	8th avg = (5.73 + 5.68 + 6.01 + 5.79)/4 = 5.80
3D	5.82	9th avg = (5.68 + 6.01 + 5.79 + 5.82)/4 = 5.83
4A	5.29	10th avg = (6.01 + 5.79 + 5.82 + 5.29)/4 = 5.73
4B	5.91	11th avg = (5.79 + 5.82 + 5.29 + 5.91)/4 = 5.70
4C	5.42	12th avg = (5.82 + 5.29 + 5.91 + 5.42)/4 = 5.61
4D	5.78	13th avg = (5.29 + 5.91 + 5.42 + 5.78)/4 = 5.60

When starting out, the first four tests (1 thru 4) will be used to determine the average. As the fifth test becomes available for plotting, a second 4-point moving average becomes available by taking the average of the 2nd thru 5th tests. This process continues as additional tests become available. Using the test data from **Table 1**, a clear pattern emerges for calculating the averages.
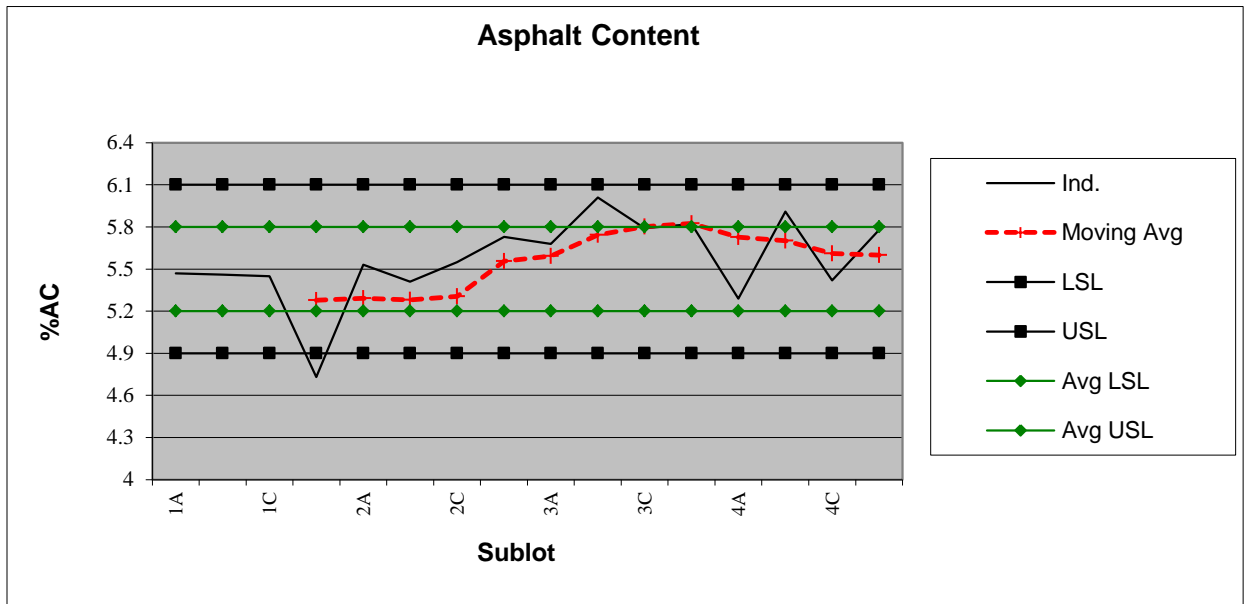
Notice how the individual test result fails in sublot 1D which exceeded the single lower specification limit (LSL). There is also a failure of the moving average in 3D. Read the specifications to determine what, if any, lower and upper specification limits exist for test results and what action is warranted when such an event occurs.

6. QUALITY LEVEL ANALYSIS

6.1. *Quality Level Analysis* is a statistical procedure that provides a method of estimating the percentage of each lot or sublot of material, product item of construction, or completed construction that may be expected to be within specified tolerance limits. This <u>percent within limits</u> is represented by the <u>unshaded</u> areas under the normal curves in **Figure 11**.

6.2. When the specifications require that the <u>percent within limits</u> be established by Quality Level Analysis, the following procedure shall apply :

Terminology:

6.2.1. x_i = the individual values under consideration

6.2.2. n = the number of individual values under consideration

6.2.3. $\overline{X}$ = the arithmetic mean or average of values under consideration. $\overline{X}$ may be expressed as $\Sigma x_i/n$, or the sum of the individual values divided by the number of individual values.

6.2.4. Q_U = Upper Quality Index. Found by subtracting the average $\overline{X}$ from the Upper Specification Limit (USL) and dividing by the sample standard deviation(s).

6.2.5. Q_L = Lower Quality Index. Found by subtracting the Lower Specification Limit (LSL) from the average $\overline{X}$ and dividing by the sample standard deviation (s).

SINGLE-LIMIT SPECIFICATION

DISTRIBUTION OF CHARACTERISTIC OF INTEREST
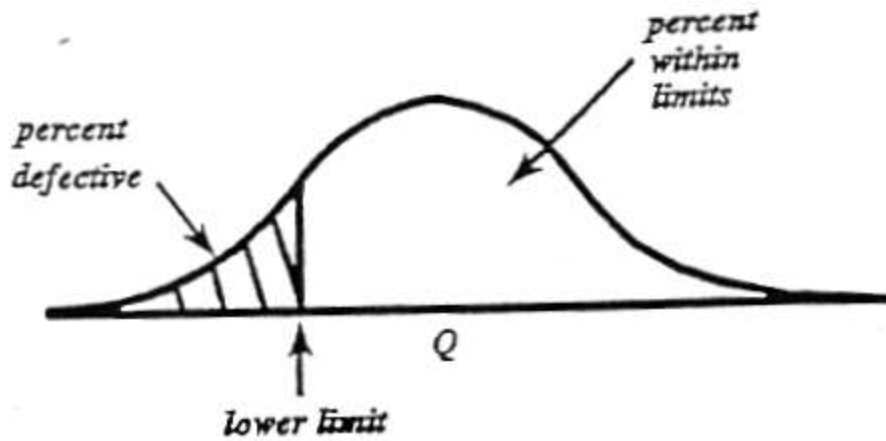
DOUBLE-LIMIT SPECIFICATION

DISTRIBUTION OF CHARACTERISTIC OF INTEREST
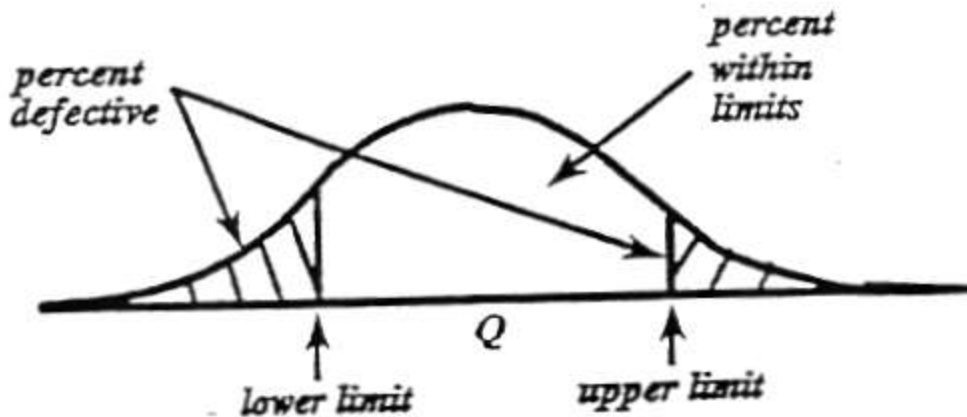
Figure 11. Concept of Percent Defective

6.3. Steps in Analysis for a double-limit specification:

6.3.1. Locate "n" sampling positions on the lot or sublot in a random manner.

6.3.2. Make a measurement at each sample position or take a test portion and make the measurement on the test portion.

6.3.3. Average all measurements to find $\bar{x}$.

6.3.4. Compute the sample standard deviation using:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

6.3.5. Find the Upper Quality Index (Q_U) by subtracting the average ($\bar{x}$) from the Upper Specification Limit (USL) and dividing the result by s.

$$Q_u = \frac{(USL - \bar{x})}{s}$$

6.3.6. Find the Lower Quality Index (Q_L) by subtracting the Lower Specification Limit (LSL) from the average ($\bar{x}$) and dividing the result by s.

$$Q_L = \frac{(\bar{x} - LSL)}{s}$$

6.3.7. Estimate the percentage that will fall below the Upper Specification limit (PWL_U). This is done by referring to **Table 2** with the computed value of Q_U and then reading the appropriate PWL_U value.

6.3.8. Estimate the percentage that will fall above the Lower Specification Limit (PWL_L).

6.3.9. Determine the Quality Level stated as percent within limits (PWL).

$$PWL = (PWL_{U +} PWL_L) - 100$$

6.4. Steps in Analysis for a single-limit specification with lower-limit specified:

6.4.1. Locate "n" sampling positions on the lot or sublot in a random manner.

6.4.2. Make a measurement at each sample position or take a test portion and make the measurement on the test portion.

6.4.3. Average all measurements to find $\bar{x}$.

6.4.4. Compute the sample standard deviation using:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

6.4.5. Find the Quality Index (Q) by subtracting the Lower Specification Limit (LSL) from the average ($\bar{X}$) and dividing the result by s.

$$Q = \frac{(\bar{x} - LSL)}{s}$$

6.4.6. Estimate the percentage that will fall above the Specification limit (PWL). This is done by referring to **Table 2** with the computed value of Q and then reading the appropriate PWL value.

6.5. Quality Level Analysis: Example Problem for double-limit specification

A contractor has run air voids tests on five lots of SM-19B. The specification limits for air voids are 4 ± 1.25 %. This sets the lower specification limit (LSL) at 2.75 % (4 - 1.25 %) air voids and the upper specification limit (USL) at 5.25 % (4 + 1.25 %) air voids. Conduct a Quality Level Analysis and compute the percent within limits.

Lot	Sublot	Percent Air Voids
1	1A	4.30
	1B	3.77
	1C	4.05
	1D	4.80
2	2A	4.90
	2B	5.07
	2C	3.82
	2D	3.53
3	3A	2.67
	3D	2.09
	3C	2.92
	3D	2.56
4	4A	2.39
	4B	2.87
	4C	5.56
	4D	4.74
5	5A	2.36
	5B	2.00
	5C	5.99
	5D	3.73

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Solution:

Lot 1: $\bar{x} = 4.23$, $s_x = 0.437$, $n = 4$

$Q_U = \dfrac{5.25 - 4.23}{0.437} = 2.33$ from **Table 2** $PWL_U = 100\%$

$Q_L = \dfrac{4.23 - 2.75}{0.437} = 3.39$ from **Table 2** $PWL_L = 100\%$

PWL = (100 + 100) - 100 = 100 %

Lot 2: $\bar{x} = 4.33$, $s_x = 0.769$, $n = 4$

$Q_U = \dfrac{5.25 - 4.33}{0.769} = 1.20$ from **Table 2** $PWL_U = 90\%$

$Q_L = \dfrac{4.33 - 2.75}{0.769} = 2.05$ from **Table 2** $PWL_L = 100\%$

PWL = (90 + 100) - 100 = 90 %

Lot 3: $\bar{x} = 2.56$, $s_x = 0.348$, $n = 4$

$Q_U = \dfrac{5.25 - 2.56}{0.348} = 7.73$ from **Table 2** $PWL_U = 100\%$

$Q_L = \dfrac{2.56 - 2.75}{0.348} = -.55$ from **Table 2** $PWL_{Table} = 68.33\%$

If Q_L is a negative number, the PWL is equal to 100 % - (value looked up in **Table 2**)

$PWL_L = (100 - 68.33) = 31.67\%$

PWL = (100 + 31.67) - 100 = 31.67 %

Lot 4: $\bar{x} = 3.89$, $s_x = 1.506$, $n = 4$

$Q_U = \dfrac{5.25 - 3.89}{1.506} = 0.90$ from **Table 2** $PWL_U = 80.0\%$

$Q_L = \dfrac{3.89 - 2.75}{1.506} = 0.76$ from **Table 2** $PWL_L = 75.33\%$

PWL = (80.0 + 75.33) - 100 = 55.33 %

Lot 5 : $\overline{x}$ = 3.52, s_x = 1.807, n = 4

$$Q_U = \frac{5.25 - 3.52}{1.807} = 0.96 \qquad \text{from } \textbf{Table 2} \text{ PWL}_U = 82.0 \text{ \%}$$

$$Q_L = \frac{3.52 - 2.75}{1.807} = 0.43 \qquad \text{from } \textbf{Table 2} \text{ PWL}_L = 64.33 \text{ \%}$$

PWL = (82.0 + 64.33) - 100 = 46.33 %

6.6. Quality Level Analysis: Example Problem for single-limit specification

A contractor has made thickness cores on three lots of concrete pavement. The lower specification limit (LSL) is 275 mm. Conduct a Quality Level Analysis and compute the percent within limits.

Lot	Sublot	Thickness (mm)
1	1A	278
	1B	274
	1C	276
	1D	280
	1E	280
2	2A	261
	2B	284
	2C	275
	2D	269
	2E	281
3	3A	293
	3D	288
	3C	297
	3D	299
	3E	290

Solution:

Lot 1 : $\overline{x}$ = 277.6, s_x = 2.608, n = 5

$$Q = \frac{277.6 - 275}{2.608} = 0.997 \qquad \text{from } \textbf{Table 2.} \text{ PWL} = 83.64 \text{ \%}$$

Lot 2 : $\overline{x}$ = 274.0, s_x = 9.274, n = 5

$$Q = \frac{274 - 275}{9.274} = -0.11 \qquad \text{from } \textbf{Table 2.} \text{ PWL}_{Table} = 53.91 \text{ \%}$$

If Q is a negative number, the PWL is equal to 100 % - (value looked up in **Table 2.**)
PWL = (100.0 - 53.91) = 46.09 %

Lot 3 : $\overline{x}$ = 293.4, s_x = 4.615, n = 5

$$Q = \frac{293.4 - 275}{4.615} = 3.99 \qquad \text{from } \textbf{Table 2.} \text{ PWL} = 100.00 \text{ \%}$$

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
0.01	50.28	50.33	50.36	50.37	50.37	50.38	50.38	50.38	50.39	50.39	50.40	50.40	50.40
0.02	50.55	50.67	50.71	50.73	50.75	50.76	50.76	50.77	50.78	50.79	50.79	50.79	50.80
0.03	50.83	51.00	51.07	51.10	51.12	51.14	51.15	51.15	51.17	51.18	51.19	51.19	51.19
0.04	51.10	51.33	51.42	51.47	51.50	51.51	51.53	51.54	51.56	51.57	51.58	51.59	51.59
0.05	51.38	51.67	51.78	51.84	51.87	51.89	51.91	51.92	51.95	51.96	51.98	51.98	51.99
0.06	51.65	52.00	52.13	52.20	52.24	52.27	52.29	52.30	52.34	52.36	52.37	52.38	52.39
0.07	51.93	52.33	52.49	52.57	52.62	52.65	52.67	52.69	52.73	52.75	52.76	52.78	52.78
0.08	52.21	52.67	52.85	52.94	52.99	53.03	53.05	53.07	53.12	53.14	53.16	53.17	53.18
0.09	52.48	53.00	53.20	53.30	53.37	53.41	53.43	53.46	53.51	53.53	53.55	53.57	53.58
0.10	52.76	53.33	53.56	53.67	53.74	53.78	53.82	53.84	53.90	53.92	53.95	53.96	53.97
0.11	53.04	53.67	53.91	54.04	54.11	54.16	54.20	54.22	54.29	54.31	54.34	54.36	54.37
0.12	53.31	54.00	54.27	54.40	54.49	54.54	54.58	54.60	54.67	54.70	54.73	54.75	54.76
0.13	53.59	54.33	54.62	54.77	54.86	54.92	54.96	54.99	55.06	55.09	55.12	55.14	55.16
0.14	53.87	54.67	54.98	55.14	55.23	55.29	55.34	55.37	55.45	55.48	55.52	55.54	55.55
0.15	54.15	55.00	55.33	55.50	55.60	55.67	55.71	55.75	55.84	55.87	55.91	55.93	55.95
0.16	54.42	55.33	55.69	55.87	55.97	56.04	56.09	56.13	56.22	56.26	56.30	56.32	56.34
0.17	54.70	55.67	56.04	56.23	56.35	56.42	56.47	56.51	56.61	56.65	56.69	56.71	56.73
0.18	54.98	56.00	56.40	56.60	56.72	56.79	56.85	56.89	56.99	57.04	57.08	57.11	57.12
0.19	55.26	56.33	56.75	56.96	57.09	57.17	57.23	57.27	57.38	57.43	57.47	57.50	57.52
0.20	55.54	56.67	57.10	57.32	57.46	57.54	57.60	57.65	57.76	57.81	57.85	57.89	57.91
0.21	55.82	57.00	57.46	57.69	57.83	57.92	57.98	58.03	58.15	58.20	58.24	58.27	58.30
0.22	56.10	57.33	57.81	58.05	58.20	58.29	58.36	58.40	58.53	58.58	58.63	58.66	58.69
0.23	56.38	57.67	58.16	58.41	58.56	58.66	58.73	58.78	58.91	58.97	59.01	59.05	59.07
0.24	56.66	58.00	58.52	58.78	58.93	59.03	59.11	59.16	59.29	59.35	59.40	59.44	59.46
0.25	56.95	58.33	58.87	59.14	59.30	59.41	59.48	59.53	59.67	59.73	59.78	59.82	59.85
0.26	57.23	58.67	59.22	59.50	59.67	59.78	59.85	59.91	60.05	60.11	60.17	60.21	60.23
0.27	57.51	59.00	59.57	59.86	60.03	60.15	60.23	60.28	60.43	60.49	60.55	60.59	60.62
0.28	57.80	59.33	59.92	60.22	60.40	60.52	60.60	60.66	60.81	60.87	60.93	60.97	61.00
0.29	58.08	59.67	60.28	60.58	60.77	60.89	60.97	61.03	61.19	61.25	61.31	61.35	61.38
0.30	58.37	60.00	60.63	60.94	61.13	61.25	61.34	61.40	61.56	61.63	61.69	61.73	61.76
0.31	58.65	60.33	60.98	61.30	61.50	61.62	61.71	61.77	61.94	62.01	62.07	62.11	62.14
0.32	58.94	60.67	61.33	61.66	61.86	61.99	62.08	62.14	62.31	62.38	62.45	62.49	62.52
0.33	59.23	61.00	61.68	62.02	62.22	62.35	62.45	62.51	62.69	62.76	62.82	62.87	62.90
0.34	59.51	61.33	62.03	62.38	62.58	62.72	62.81	62.88	63.06	63.13	63.20	63.25	63.28
0.35	59.80	61.67	62.38	62.73	62.94	63.08	63.18	63.25	63.43	63.51	63.57	63.62	63.65
0.36	60.09	62.00	62.72	63.09	63.31	63.45	63.54	63.62	63.80	63.88	63.95	63.99	64.03
0.37	60.38	62.33	63.07	63.45	63.67	63.81	63.91	63.98	64.17	64.25	64.32	64.37	64.40
0.38	60.67	62.67	63.42	63.80	64.02	64.17	64.27	64.35	64.54	64.62	64.69	64.74	64.77
0.39	60.97	63.00	63.77	64.16	64.38	64.53	64.63	64.71	64.90	64.98	65.06	65.11	65.14
0.40	61.26	63.33	64.12	64.51	64.74	64.89	65.00	65.07	65.27	65.35	65.42	65.47	65.51
0.41	61.55	63.67	64.46	64.86	65.10	65.25	65.36	65.43	65.63	65.72	65.79	65.84	65.88
0.42	61.85	64.00	64.81	65.21	65.45	65.61	65.71	65.79	66.00	66.08	66.15	66.21	66.24
0.43	62.15	64.33	65.15	65.57	65.81	65.96	66.07	66.15	66.36	66.44	66.52	66.57	66.61
0.44	62.44	64.67	65.50	65.92	66.16	66.32	66.43	66.51	66.72	66.80	66.88	66.93	66.97

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.45	62.74	65.00	65.84	66.27	66.51	66.67	66.79	66.87	67.08	67.16	67.24	67.29	67.33
0.46	63.04	65.33	66.19	66.62	66.87	67.03	67.14	67.22	67.43	67.52	67.60	67.65	67.69
0.47	63.34	65.67	66.53	66.96	67.22	67.38	67.49	67.58	67.79	67.88	67.96	68.01	68.05
0.48	63.65	66.00	66.88	67.31	67.57	67.73	67.85	67.93	68.15	68.23	68.31	68.37	68.40
0.49	63.95	66.33	67.22	67.66	67.92	68.08	68.20	68.28	68.50	68.59	68.67	68.72	68.76
0.50	64.25	66.67	67.56	68.00	68.26	68.43	68.55	68.63	68.85	68.94	69.02	69.07	69.11
0.51	64.56	67.00	67.90	68.35	68.61	68.78	68.90	68.98	69.20	69.29	69.37	69.43	69.46
0.52	64.87	67.33	68.24	68.69	68.96	69.13	69.24	69.33	69.55	69.64	69.72	69.77	69.81
0.53	65.18	67.67	68.58	69.04	69.30	69.47	69.59	69.68	69.90	69.99	70.07	70.12	70.16
0.54	65.49	68.00	68.92	69.38	69.64	69.82	69.93	70.02	70.24	70.33	70.41	70.47	70.51
0.55	65.80	68.33	69.26	69.72	69.99	70.16	70.28	70.36	70.59	70.68	70.76	70.81	70.85
0.56	66.12	68.67	69.60	70.06	70.33	70.50	70.62	70.71	70.93	71.02	71.10	71.15	71.19
0.57	66.43	69.00	69.94	70.40	70.67	70.84	70.96	71.05	71.27	71.36	71.44	71.49	71.53
0.58	66.75	69.33	70.27	70.74	71.01	71.18	71.30	71.39	71.61	71.70	71.78	71.83	71.87
0.59	67.07	69.67	70.61	71.07	71.34	71.52	71.64	71.72	71.95	72.04	72.11	72.17	72.21
0.60	67.39	70.00	70.95	71.41	71.68	71.85	71.97	72.06	72.28	72.37	72.45	72.50	72.54
0.61	67.72	70.33	71.28	71.75	72.02	72.19	72.31	72.40	72.61	72.70	72.78	72.84	72.87
0.62	68.04	70.67	71.61	72.08	72.35	72.52	72.64	72.73	72.95	73.04	73.11	73.17	73.20
0.63	68.37	71.00	71.95	72.41	72.68	72.85	72.97	73.06	73.28	73.37	73.44	73.50	73.53
0.64	68.70	71.33	72.28	72.74	73.01	73.18	73.30	73.39	73.61	73.69	73.77	73.82	73.86
0.65	69.03	71.67	72.61	73.08	73.34	73.51	73.63	73.72	73.93	74.02	74.10	74.15	74.18
0.66	69.37	72.00	72.94	73.40	73.67	73.84	73.96	74.04	74.26	74.34	74.42	74.47	74.51
0.67	69.70	72.33	73.27	73.73	74.00	74.17	74.28	74.37	74.58	74.67	74.74	74.79	74.83
0.68	70.04	72.67	73.60	74.06	74.32	74.49	74.61	74.69	74.90	74.99	75.06	75.11	75.14
0.69	70.39	73.00	73.93	74.39	74.65	74.81	74.93	75.01	75.22	75.30	75.38	75.43	75.46
0.70	70.73	73.33	74.26	74.71	74.97	75.14	75.25	75.33	75.54	75.62	75.69	75.74	75.77
0.71	71.08	73.67	74.59	75.04	75.29	75.46	75.57	75.65	75.85	75.94	76.01	76.05	76.09
0.72	71.43	74.00	74.91	75.36	75.61	75.77	75.89	75.97	76.17	76.25	76.32	76.36	76.40
0.73	71.78	74.33	75.24	75.68	75.93	76.09	76.20	76.28	76.48	76.56	76.63	76.67	76.70
0.74	72.14	74.67	75.56	76.00	76.25	76.41	76.51	76.59	76.79	76.87	76.93	76.98	77.01
0.75	72.50	75.00	75.89	76.32	76.56	76.72	76.83	76.90	77.10	77.17	77.24	77.28	77.31
0.76	72.87	75.33	76.21	76.63	76.88	77.03	77.14	77.21	77.40	77.48	77.54	77.58	77.61
0.77	73.24	75.67	76.53	76.95	77.19	77.34	77.44	77.52	77.70	77.78	77.84	77.88	77.91
0.78	73.61	76.00	76.85	77.26	77.50	77.65	77.75	77.82	78.01	78.08	78.14	78.18	78.21
0.79	73.98	76.33	77.17	77.58	77.81	77.96	78.06	78.13	78.30	78.37	78.43	78.47	78.50
0.80	74.36	76.67	77.49	77.89	78.12	78.26	78.36	78.43	78.60	78.67	78.73	78.77	78.79
0.81	74.75	77.00	77.81	78.20	78.42	78.56	78.66	78.73	78.90	78.96	79.02	79.06	79.08
0.82	75.14	77.33	78.13	78.51	78.73	78.86	78.96	79.02	79.19	79.25	79.31	79.35	79.37
0.83	75.53	77.67	78.44	78.82	79.03	79.16	79.25	79.32	79.48	79.54	79.60	79.63	79.65
0.84	75.93	78.00	78.76	79.12	79.33	79.46	79.55	79.61	79.77	79.83	79.88	79.91	79.94
0.85	76.33	78.33	79.07	79.43	79.63	79.76	79.84	79.90	80.06	80.11	80.16	80.20	80.22
0.86	76.74	78.67	79.38	79.73	79.93	80.05	80.13	80.19	80.34	80.40	80.44	80.47	80.49
0.87	77.16	79.00	79.69	80.03	80.22	80.34	80.42	80.48	80.62	80.68	80.72	80.75	80.77
0.88	77.58	79.33	80.00	80.33	80.52	80.63	80.71	80.77	80.90	80.95	81.00	81.02	81.04
0.89	78.01	79.67	80.31	80.63	80.81	80.92	81.00	81.05	81.18	81.23	81.27	81.30	81.31

Revised 2014

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
0.90	78.45	80.00	80.62	80.93	81.10	81.21	81.28	81.33	81.46	81.50	81.54	81.57	81.58
0.91	78.89	80.33	80.93	81.22	81.39	81.49	81.56	81.61	81.73	81.77	81.81	81.83	81.85
0.92	79.34	80.67	81.23	81.51	81.67	81.77	81.84	81.89	82.00	82.04	82.08	82.10	82.11
0.93	79.81	81.00	81.54	81.81	81.96	82.05	82.12	82.16	82.27	82.31	82.34	82.36	82.37
0.94	80.27	81.33	81.84	82.10	82.24	82.33	82.39	82.44	82.54	82.57	82.60	82.62	82.63
0.95	80.75	81.67	82.14	82.39	82.52	82.61	82.67	82.71	82.80	82.84	82.86	82.88	82.89
0.96	81.25	82.00	82.45	82.67	82.80	82.88	82.94	82.97	83.06	83.10	83.12	83.13	83.14
0.97	81.75	82.33	82.75	82.96	83.08	83.15	83.21	83.24	83.32	83.35	83.37	83.39	83.39
0.98	82.26	82.67	83.04	83.24	83.35	83.43	83.47	83.51	83.58	83.61	83.63	83.64	83.64
0.99	82.79	83.00	83.34	83.52	83.63	83.69	83.74	83.77	83.84	83.86	83.88	83.88	83.89
1.00	83.33	83.33	83.64	83.80	83.90	83.96	84.00	84.03	84.09	84.11	84.12	84.13	84.13
1.01	83.89	83.67	83.93	84.08	84.17	84.22	84.26	84.28	84.34	84.36	84.37	84.37	84.38
1.02	84.47	84.00	84.22	84.36	84.44	84.49	84.52	84.54	84.59	84.60	84.61	84.62	84.62
1.03	85.07	84.33	84.52	84.63	84.70	84.75	84.77	84.79	84.83	84.85	84.85	84.85	84.85
1.04	85.69	84.67	84.81	84.91	84.97	85.00	85.03	85.04	85.08	85.09	85.09	85.09	85.09
1.05	86.34	85.00	85.09	85.18	85.23	85.26	85.28	85.29	85.32	85.33	85.33	85.32	85.32
1.06	87.02	85.33	85.38	85.45	85.49	85.51	85.53	85.54	85.56	85.56	85.56	85.55	85.55
1.07	87.73	85.67	85.67	85.71	85.74	85.76	85.78	85.78	85.80	85.80	85.79	85.78	85.78
1.08	88.49	86.00	85.95	85.98	86.00	86.01	86.02	86.03	86.03	86.03	86.02	86.01	86.00
1.09	89.29	86.33	86.24	86.24	86.25	86.26	86.27	86.27	86.26	86.26	86.25	86.23	86.23
1.10	90.16	86.67	86.52	86.50	86.51	86.51	86.51	86.50	86.49	86.48	86.47	86.46	86.45
1.11	91.11	87.00	86.80	86.76	86.75	86.75	86.74	86.74	86.72	86.71	86.69	86.68	86.66
1.12	92.18	87.33	87.07	87.02	87.00	86.99	86.98	86.97	86.95	86.93	86.91	86.89	86.88
1.13	93.40	87.67	87.35	87.28	87.25	87.23	87.21	87.20	87.17	87.15	87.13	87.11	87.09
1.14	94.92	88.00	87.63	87.53	87.49	87.46	87.45	87.43	87.39	87.37	87.34	87.32	87.30
1.15	97.13	88.33	87.90	87.78	87.73	87.70	87.68	87.66	87.61	87.58	87.55	87.53	87.51
1.16	100.00	88.67	88.17	88.03	87.97	87.93	87.90	87.88	87.82	87.79	87.76	87.74	87.72
1.17	100.00	89.00	88.44	88.28	88.21	88.16	88.13	88.10	88.04	88.00	87.97	87.94	87.92
1.18	100.00	89.33	88.71	88.53	88.44	88.39	88.35	88.32	88.25	88.21	88.18	88.15	88.12
1.19	100.00	89.67	88.98	88.77	88.67	88.61	88.57	88.54	88.46	88.42	88.38	88.35	88.32
1.20	100.00	90.00	89.24	89.01	88.90	88.83	88.79	88.76	88.66	88.62	88.58	88.54	88.52
1.21	100.00	90.33	89.50	89.25	89.13	89.06	89.00	88.97	88.87	88.82	88.78	88.74	88.71
1.22	100.00	90.67	89.77	89.49	89.35	89.27	89.22	89.18	89.07	89.02	88.97	88.93	88.91
1.23	100.00	91.00	90.03	89.72	89.58	89.49	89.43	89.39	89.27	89.22	89.16	89.12	89.09
1.24	100.00	91.33	90.28	89.96	89.80	89.70	89.64	89.59	89.47	89.41	89.36	89.31	89.28
1.25	100.00	91.67	90.54	90.19	90.02	89.91	89.85	89.79	89.66	89.60	89.54	89.50	89.47
1.26	100.00	92.00	90.79	90.42	90.23	90.12	90.05	90.00	89.85	89.79	89.73	89.68	89.65
1.27	100.00	92.33	91.04	90.64	90.45	90.33	90.25	90.19	90.04	89.98	89.91	89.87	89.83
1.28	100.00	92.67	91.29	90.87	90.66	90.53	90.45	90.39	90.23	90.16	90.10	90.05	90.01
1.29	100.00	93.00	91.54	91.09	90.87	90.74	90.65	90.58	90.42	90.34	90.28	90.22	90.18
1.30	100.00	93.33	91.79	91.31	91.07	90.94	90.84	90.78	90.60	90.52	90.45	90.40	90.36
1.31	100.00	93.67	92.03	91.52	91.28	91.13	91.04	90.97	90.78	90.70	90.63	90.57	90.53
1.32	100.00	94.00	92.27	91.74	91.48	91.33	91.23	91.15	90.96	90.88	90.80	90.74	90.70
1.33	100.00	94.33	92.51	91.95	91.68	91.52	91.41	91.34	91.14	91.05	90.97	90.91	90.87
1.34	100.00	94.67	92.75	92.16	91.88	91.71	91.60	91.52	91.31	91.22	91.14	91.08	91.03

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Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.35	100.00	95.00	92.98	92.37	92.08	91.90	91.78	91.70	91.48	91.39	91.31	91.24	91.19
1.36	100.00	95.33	93.21	92.58	92.27	92.09	91.96	91.88	91.65	91.56	91.47	91.40	91.35
1.37	100.00	95.67	93.44	92.78	92.46	92.27	92.14	92.05	91.82	91.72	91.63	91.56	91.51
1.38	100.00	96.00	93.67	92.98	92.65	92.45	92.32	92.23	91.99	91.88	91.79	91.72	91.67
1.39	100.00	96.33	93.90	93.18	92.83	92.63	92.49	92.40	92.15	92.04	91.95	91.88	91.82
1.40	100.00	96.67	94.12	93.37	93.02	92.81	92.67	92.56	92.31	92.20	92.10	92.03	91.98
1.41	100.00	97.00	94.34	93.57	93.20	92.98	92.83	92.73	92.47	92.36	92.26	92.18	92.13
1.42	100.00	97.33	94.56	93.76	93.38	93.15	93.00	92.90	92.63	92.51	92.41	92.33	92.27
1.43	100.00	97.67	94.77	93.95	93.55	93.32	93.17	93.06	92.78	92.66	92.56	92.48	92.42
1.44	100.00	98.00	94.98	94.13	93.73	93.49	93.33	93.22	92.93	92.81	92.70	92.62	92.56
1.45	100.00	98.33	95.19	94.32	93.90	93.65	93.49	93.37	93.08	92.96	92.85	92.76	92.70
1.46	100.00	98.67	95.40	94.50	94.07	93.81	93.65	93.53	93.23	93.10	92.99	92.90	92.84
1.47	100.00	99.00	95.61	94.67	94.23	93.97	93.80	93.68	93.37	93.25	93.13	93.04	92.98
1.48	100.00	99.33	95.81	94.85	94.40	94.13	93.96	93.83	93.52	93.39	93.27	93.18	93.12
1.49	100.00	99.67	96.01	95.02	94.56	94.29	94.11	93.98	93.66	93.52	93.40	93.31	93.25
1.50	100.00	100.00	96.20	95.19	94.72	94.44	94.26	94.13	93.80	93.66	93.54	93.45	93.38
1.51	100.00	100.00	96.39	95.36	94.87	94.59	94.40	94.27	93.94	93.80	93.67	93.58	93.51
1.52	100.00	100.00	96.58	95.53	95.03	94.74	94.55	94.41	94.07	93.93	93.80	93.71	93.64
1.53	100.00	100.00	96.77	95.69	95.18	94.88	94.69	94.55	94.20	94.06	93.93	93.83	93.76
1.54	100.00	100.00	96.95	95.85	95.33	95.03	94.83	94.69	94.33	94.19	94.05	93.96	93.89
1.55	100.00	100.00	97.13	96.00	95.48	95.17	94.97	94.82	94.46	94.31	94.18	94.08	94.01
1.56	100.00	100.00	97.31	96.16	95.62	95.31	95.10	94.95	94.59	94.44	94.30	94.20	94.13
1.57	100.00	100.00	97.48	96.31	95.76	95.44	95.23	95.08	94.71	94.56	94.42	94.32	94.25
1.58	100.00	100.00	97.65	96.46	95.90	95.58	95.36	95.21	94.84	94.68	94.54	94.44	94.36
1.59	100.00	100.00	97.81	96.60	96.04	95.71	95.49	95.34	94.96	94.80	94.66	94.55	94.48
1.60	100.00	100.00	97.97	96.75	96.17	95.84	95.62	95.46	95.08	94.92	94.77	94.67	94.59
1.61	100.00	100.00	98.13	96.89	96.31	95.97	95.74	95.59	95.19	95.03	94.88	94.78	94.70
1.62	100.00	100.00	98.28	97.03	96.43	96.09	95.86	95.70	95.31	95.14	94.99	94.89	94.81
1.63	100.00	100.00	98.43	97.16	96.56	96.21	95.98	95.82	95.42	95.25	95.10	94.99	94.92
1.64	100.00	100.00	98.58	97.29	96.69	96.33	96.10	95.94	95.53	95.36	95.21	95.10	95.02
1.65	100.00	100.00	98.72	97.42	96.81	96.45	96.22	96.05	95.64	95.47	95.32	95.21	95.13
1.66	100.00	100.00	98.85	97.55	96.93	96.57	96.33	96.16	95.75	95.57	95.42	95.31	95.23
1.67	100.00	100.00	98.98	97.67	97.05	96.68	96.44	96.27	95.85	95.68	95.52	95.41	95.33
1.68	100.00	100.00	99.11	97.79	97.16	96.79	96.55	96.38	95.95	95.78	95.62	95.51	95.43
1.69	100.00	100.00	99.23	97.91	97.27	96.90	96.66	96.48	96.06	95.88	95.72	95.61	95.53
1.70	100.00	100.00	99.34	98.02	97.38	97.01	96.76	96.59	96.16	95.98	95.82	95.70	95.62
1.71	100.00	100.00	99.45	98.13	97.49	97.11	96.86	96.69	96.25	96.07	95.91	95.80	95.71
1.72	100.00	100.00	99.55	98.24	97.59	97.21	96.97	96.79	96.35	96.17	96.01	95.89	95.81
1.73	100.00	100.00	99.64	98.34	97.70	97.31	97.06	96.89	96.44	96.26	96.10	95.98	95.90
1.74	100.00	100.00	99.73	98.45	97.80	97.41	97.16	96.98	96.54	96.35	96.19	96.07	95.99
1.75	100.00	100.00	99.81	98.55	97.89	97.51	97.25	97.07	96.63	96.44	96.28	96.16	96.07
1.76	100.00	100.00	99.88	98.64	97.99	97.60	97.35	97.17	96.72	96.53	96.37	96.24	96.16
1.77	100.00	100.00	99.94	98.73	98.08	97.69	97.44	97.26	96.80	96.62	96.45	96.33	96.24
1.78	100.00	100.00	99.98	98.82	98.17	97.78	97.53	97.34	96.89	96.70	96.53	96.41	96.33
1.79	100.00	100.00	100.00	98.91	98.26	97.87	97.61	97.43	96.97	96.79	96.62	96.49	96.41

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
1.80	100.00	100.00	100.00	98.99	98.35	97.96	97.70	97.51	97.06	96.87	96.70	96.57	96.49
1.81	100.00	100.00	100.00	99.07	98.43	98.04	97.78	97.60	97.14	96.95	96.78	96.65	96.57
1.82	100.00	100.00	100.00	99.15	98.51	98.12	97.86	97.68	97.21	97.02	96.85	96.73	96.64
1.83	100.00	100.00	100.00	99.22	98.59	98.20	97.94	97.75	97.29	97.10	96.93	96.81	96.72
1.84	100.00	100.00	100.00	99.29	98.66	98.28	98.02	97.83	97.37	97.18	97.01	96.88	96.79
1.85	100.00	100.00	100.00	99.36	98.74	98.35	98.09	97.91	97.44	97.25	97.08	96.95	96.87
1.86	100.00	100.00	100.00	99.43	98.81	98.42	98.16	97.98	97.52	97.32	97.15	97.03	96.94
1.87	100.00	100.00	100.00	99.49	98.88	98.49	98.24	98.05	97.59	97.39	97.22	97.10	97.01
1.88	100.00	100.00	100.00	99.54	98.94	98.56	98.30	98.12	97.66	97.46	97.29	97.17	97.08
1.89	100.00	100.00	100.00	99.60	99.01	98.63	98.37	98.19	97.72	97.53	97.36	97.23	97.15
1.90	100.00	100.00	100.00	99.65	99.07	98.69	98.44	98.25	97.79	97.60	97.43	97.30	97.21
1.91	100.00	100.00	100.00	99.70	99.13	98.76	98.50	98.32	97.86	97.66	97.49	97.37	97.28
1.92	100.00	100.00	100.00	99.74	99.19	98.82	98.56	98.38	97.92	97.73	97.55	97.43	97.34
1.93	100.00	100.00	100.00	99.78	99.24	98.88	98.63	98.44	97.98	97.79	97.62	97.49	97.40
1.94	100.00	100.00	100.00	99.82	99.30	98.93	98.68	98.50	98.04	97.85	97.68	97.55	97.46
1.95	100.00	100.00	100.00	99.85	99.35	98.99	98.74	98.56	98.10	97.91	97.74	97.61	97.52
1.96	100.00	100.00	100.00	99.88	99.40	99.04	98.80	98.62	98.16	97.97	97.80	97.67	97.58
1.97	100.00	100.00	100.00	99.91	99.44	99.09	98.85	98.67	98.22	98.03	97.86	97.73	97.64
1.98	100.00	100.00	100.00	99.93	99.49	99.14	98.90	98.73	98.27	98.08	97.91	97.79	97.70
1.99	100.00	100.00	100.00	99.95	99.53	99.19	98.95	98.78	98.33	98.14	97.97	97.84	97.75
2.00	100.00	100.00	100.00	99.97	99.57	99.24	99.00	98.83	98.38	98.19	98.02	97.90	97.81
2.01	100.00	100.00	100.00	99.98	99.61	99.28	99.05	98.88	98.43	98.24	98.07	97.95	97.86
2.02	100.00	100.00	100.00	99.99	99.64	99.33	99.10	98.93	98.48	98.29	98.13	98.00	97.91
2.03	100.00	100.00	100.00	100.00	99.68	99.37	99.14	98.97	98.53	98.34	98.18	98.05	97.96
2.04	100.00	100.00	100.00	100.00	99.71	99.41	99.18	99.02	98.58	98.39	98.23	98.10	98.01
2.05	100.00	100.00	100.00	100.00	99.74	99.45	99.23	99.06	98.63	98.44	98.27	98.15	98.06
2.06	100.00	100.00	100.00	100.00	99.77	99.48	99.27	99.10	98.67	98.49	98.32	98.20	98.11
2.07	100.00	100.00	100.00	100.00	99.79	99.52	99.30	99.14	98.72	98.53	98.37	98.24	98.16
2.08	100.00	100.00	100.00	100.00	99.82	99.55	99.34	99.18	98.76	98.58	98.41	98.29	98.21
2.09	100.00	100.00	100.00	100.00	99.84	99.58	99.38	99.22	98.80	98.62	98.46	98.34	98.25
2.10	100.00	100.00	100.00	100.00	99.86	99.61	99.41	99.26	98.84	98.66	98.50	98.38	98.29
2.11	100.00	100.00	100.00	100.00	99.88	99.64	99.45	99.29	98.88	98.70	98.54	98.42	98.34
2.12	100.00	100.00	100.00	100.00	99.90	99.67	99.48	99.33	98.92	98.74	98.58	98.46	98.38
2.13	100.00	100.00	100.00	100.00	99.92	99.70	99.51	99.36	98.96	98.78	98.62	98.50	98.42
2.14	100.00	100.00	100.00	100.00	99.93	99.72	99.54	99.39	99.00	98.82	98.66	98.54	98.46
2.15	100.00	100.00	100.00	100.00	99.94	99.74	99.57	99.42	99.03	98.86	98.70	98.58	98.50
2.16	100.00	100.00	100.00	100.00	99.95	99.77	99.59	99.45	99.07	98.90	98.74	98.62	98.54
2.17	100.00	100.00	100.00	100.00	99.96	99.79	99.62	99.48	99.10	98.93	98.78	98.66	98.58
2.18	100.00	100.00	100.00	100.00	99.97	99.81	99.64	99.51	99.13	98.97	98.81	98.70	98.61
2.19	100.00	100.00	100.00	100.00	99.98	99.83	99.67	99.54	99.17	99.00	98.85	98.73	98.65
2.20	100.00	100.00	100.00	100.00	99.99	99.84	99.69	99.56	99.20	99.03	98.88	98.77	98.69
2.21	100.00	100.00	100.00	100.00	99.99	99.86	99.71	99.59	99.23	99.06	98.91	98.80	98.72
2.22	100.00	100.00	100.00	100.00	99.99	99.87	99.73	99.61	99.26	99.10	98.95	98.83	98.75
2.23	100.00	100.00	100.00	100.00	100.00	99.89	99.75	99.63	99.29	99.13	98.98	98.87	98.79
2.24	100.00	100.00	100.00	100.00	100.00	99.90	99.77	99.66	99.31	99.15	99.01	98.90	98.82

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
2.25	100.00	100.00	100.00	100.00	100.00	99.91	99.79	99.68	99.34	99.18	99.04	98.93	98.85
2.26	100.00	100.00	100.00	100.00	100.00	99.92	99.80	99.70	99.37	99.21	99.07	98.96	98.88
2.27	100.00	100.00	100.00	100.00	100.00	99.93	99.82	99.71	99.39	99.24	99.10	98.99	98.91
2.28	100.00	100.00	100.00	100.00	100.00	99.94	99.83	99.73	99.42	99.26	99.12	99.02	98.94
2.29	100.00	100.00	100.00	100.00	100.00	99.95	99.85	99.75	99.44	99.29	99.15	99.05	98.97
2.30	100.00	100.00	100.00	100.00	100.00	99.96	99.86	99.77	99.46	99.32	99.18	99.07	99.00
2.31	100.00	100.00	100.00	100.00	100.00	99.96	99.87	99.78	99.48	99.34	99.20	99.10	99.03
2.32	100.00	100.00	100.00	100.00	100.00	99.97	99.89	99.80	99.51	99.36	99.23	99.13	99.05
2.33	100.00	100.00	100.00	100.00	100.00	99.98	99.90	99.81	99.53	99.39	99.25	99.15	99.08
2.34	100.00	100.00	100.00	100.00	100.00	99.98	99.91	99.82	99.55	99.41	99.28	99.18	99.10
2.35	100.00	100.00	100.00	100.00	100.00	99.98	99.92	99.84	99.57	99.43	99.30	99.20	99.13
2.36	100.00	100.00	100.00	100.00	100.00	99.99	99.92	99.85	99.58	99.45	99.32	99.22	99.15
2.37	100.00	100.00	100.00	100.00	100.00	99.99	99.93	99.86	99.60	99.47	99.34	99.25	99.18
2.38	100.00	100.00	100.00	100.00	100.00	99.99	99.94	99.87	99.62	99.49	99.37	99.27	99.20
2.39	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.88	99.64	99.51	99.39	99.29	99.22
2.40	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.89	99.65	99.53	99.41	99.31	99.25
2.41	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.90	99.67	99.55	99.43	99.33	99.27
2.42	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.91	99.68	99.56	99.44	99.35	99.29
2.43	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.92	99.70	99.58	99.46	99.37	99.31
2.44	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.92	99.71	99.60	99.48	99.39	99.33
2.45	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.93	99.73	99.61	99.50	99.41	99.35
2.46	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.94	99.74	99.63	99.52	99.43	99.37
2.47	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.94	99.75	99.64	99.53	99.45	99.38
2.48	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.95	99.76	99.66	99.55	99.46	99.40
2.49	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.95	99.77	99.67	99.56	99.48	99.42
2.50	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.79	99.68	99.58	99.50	99.44
2.51	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.80	99.70	99.59	99.51	99.45
2.52	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.81	99.71	99.61	99.53	99.47
2.53	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.82	99.72	99.62	99.54	99.49
2.54	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.83	99.73	99.63	99.56	99.50
2.55	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.83	99.74	99.65	99.57	99.52
2.56	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.84	99.75	99.66	99.59	99.53
2.57	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.85	99.76	99.67	99.60	99.54
2.58	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.86	99.77	99.68	99.61	99.56
2.59	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.87	99.78	99.70	99.62	99.57
2.60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.87	99.79	99.71	99.64	99.59
2.61	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.88	99.80	99.72	99.65	99.60
2.62	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.89	99.81	99.73	99.66	99.61
2.63	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.89	99.82	99.74	99.67	99.62
2.64	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.90	99.83	99.75	99.68	99.63
2.65	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.91	99.84	99.76	99.69	99.65
2.66	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.91	99.84	99.77	99.70	99.66
2.67	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.92	99.85	99.78	99.71	99.67
2.68	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.92	99.86	99.78	99.72	99.68
2.69	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.93	99.86	99.79	99.73	99.69

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
2.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.93	99.87	99.80	99.74	99.70
2.71	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.94	99.88	99.81	99.75	99.71
2.72	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.94	99.88	99.82	99.76	99.72
2.73	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.94	99.89	99.82	99.77	99.73
2.74	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.89	99.83	99.78	99.73
2.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.90	99.84	99.78	99.74
2.76	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.90	99.84	99.79	99.75
2.77	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.91	99.85	99.80	99.76
2.78	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.91	99.86	99.81	99.77
2.79	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.96	99.92	99.86	99.81	99.77
2.80	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.92	99.87	99.82	99.78
2.81	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.93	99.87	99.83	99.79
2.82	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.93	99.88	99.83	99.80
2.83	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.93	99.88	99.84	99.80
2.84	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.97	99.94	99.89	99.84	99.81
2.85	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.94	99.89	99.85	99.82
2.86	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.94	99.90	99.86	99.82
2.87	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.95	99.90	99.86	99.83
2.88	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.95	99.91	99.87	99.83
2.89	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.95	99.91	99.87	99.84
2.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.95	99.91	99.88	99.84
2.91	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.92	99.88	99.85
2.92	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.92	99.88	99.86
2.93	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.92	99.89	99.86
2.94	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.93	99.89	99.87
2.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.93	99.90	99.87
2.96	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.93	99.90	99.87
2.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.94	99.90	99.88
2.98	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.94	99.91	99.88
2.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.94	99.91	99.89
3.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.92	99.89
3.01	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.95	99.92	99.89
3.02	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.95	99.92	99.90
3.03	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.95	99.93	99.90
3.04	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.95	99.93	99.91
3.05	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.96	99.93	99.91
3.06	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.96	99.93	99.91
3.07	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.96	99.94	99.92
3.08	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.94	99.92
3.09	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.94	99.92
3.10	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.94	99.92
3.11	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.93
3.12	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.93
3.13	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.93
3.14	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.93

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
3.15	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.95	99.94
3.16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.94
3.17	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.94
3.18	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.94
3.19	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.95
3.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.95
3.21	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96	99.95
3.22	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97	99.95
3.23	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97	99.95
3.24	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.97	99.96
3.25	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.98	99.97	99.96
3.26	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.96
3.27	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.96
3.28	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.97	99.96
3.29	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96
3.30	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.96
3.31	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.32	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.33	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.34	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.35	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.36	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.37	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.38	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.97
3.39	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98	99.98
3.40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.41	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.42	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.43	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.44	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99	99.98
3.45	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98
3.46	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98
3.47	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98
3.48	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98
3.49	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98
3.50	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.98
3.51	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.52	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.53	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.54	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.55	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.56	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.57	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.58	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.59	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99

Table 2 for Estimation of Lot Percent Within Limits
Variability Unknown Procedure
Standard Deviation Method

Quality Index Q_U or Q_L	Percent Within Limits for Selected Sample Sizes												
	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=15	N=20	N=30	N=50	N=100
3.60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.61	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99	99.99
3.62	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.63	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.64	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.65	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.66	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.67	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.68	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.69	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.71	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.72	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.73	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.74	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.99
3.76	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

The estimates of lot percent within limits (PWL) provided in the tables are obtained by numerically integrating the beta distribution function corresponding to Quality Index (Q) and Sample Size (N).

To find PWL from the tables, compute Q from the sample mean and sample standard deviation with unknown population variability, and the lower or upper specification limits.

To find the PWL for a negative Quality Index, first get the PWL for the positive value of the Quality Index from the tables and subtract the result from 100.

5.2.2.2. RANDOM SAMPLING

1. SCOPE

This method covers procedures for securing random samples from a lot by the use of random numbers obtained from tables or generated by other methods.

Nothing in this method is intended to preclude additional testing if failing or suspect materials or construction is encountered. Testing that is additional to the scheduled testing should occur immediately if failing test results occur or if materials or work appears to be substandard.

2. DEFINITIONS

2.1. Lot: An isolated or defined quantity of material from a single source or a measured amount of construction assumed to be produced by the same process. Specified amounts of asphalt concrete mix, a stockpile of aggregates, or linear feet of roadway constructed in a day are examples of a lot.

2.1.1. Sublot: A portion of a lot. When it is not convenient to sample the entire lot, such as a specified amount of hot mix, then it can be divided into equal sized sublots. The sublots, when combined, would constitute the entire lot.

2.1.2. Random: Without aim or pattern, depending entirely on chance alone (not to be construed as haphazard).

2.1.3. Sample: A small part of a lot or sublot which represents the whole. A sample may be made up of one or more increments or test portions.

2.1.4. Random number: A number selected entirely by chance. Random numbers may be generated electronically such as with a random number function on a calculator or spreadsheet or selected from a table of random numbers (See **Table 1**).

3. USE OF RANDOM NUMBERS IN SAMPLING

3.1. Most sampling and testing for construction materials should be randomized to prevent any unintentional bias of the results. Randomization of sampling times or locations is accomplished by using a set of random numbers to determine the time or location for the sample. A table of random numbers is included below.

3.1.1. Example 1: Determining when to sample.

3.1.1.1. As an example, assume ten trucks carrying equal loads are going to be used to deliver concrete during a bridge deck placement. Select which truck to sample for compressive strength cylinders to be molded.

In cases such as number of trucks or tons of production, etc. round up to the next whole number because there will not be a truck "0". In cases involving things such as stationing there is a zero point so rounding to the nearest number may be justified.

Generate a random number using one of the methods below. Use .456 in this example. Multiply .456 by 10 (the number of trucks) for a result of 4.56. Round this result up to a whole number, 5 in this case. Take the concrete sample from the fifth truck.

This method can be used to select a time of day or the day of the week. If production was to occur during an 8 hour day, multiply 8 by the selected random number, .456, to obtain a result of 3.648. If rounding is used the sample should be taken in the fourth hour of production. Refinement could be used to select a time down to the nearest minute if needed by using the integer, three in this case, as the hour and then multiplying the decimal by 60 to obtain the minute, 60 X .648 or $\pm$ 39 in this case. Sampling would occur three hours thirty nine minutes into production. Use the number seven multiplied by a random number to determine a day of the week. Sampling during production according to units, such as tons of material produced, can be handled in the same fashion. Multiply the lot or sublot size, as required, by the random number selected.

3.2. Example 2: Determining location for sampling.

3.2.1. Given random numbers selected:

X	Y
0.338	0.922
0.763	0.198
0.043	0.737
0.810	0.747

ENGLISH EXAMPLE:

Sampling a large lot may require division into sublots to insure all portions of a lot are represented. Stratification into sublots is accomplished by dividing the "Lot" material (in this case, a mile of construction or 5280 feet, 12'6" wide) into "four sublots" (each of 1320' or 1/4 mile).

To locate a sample point station in sublot No. 1, the length of that sublot is multiplied by the "X" coordinate for the "sublot" and the product added to the beginning station for that sublot.

Starting Station = 486 + 15   486 + 15
$(X_1)(1320) = (0.338) (1320) = 446'$ 4 + 46
Sample Station =   490 + 61

The sample point distance from the base line (generally centerline or the edge of pavement) is determined by multiplying Y_1 by the available width, in this case, 12.5 feet.

$(Y_1)(12.5') = (0.922) (12.5') = 11.5'$ from base line.

Thus the sample location is Sta. 490 + 61, 11.5' from base line. Keeping in mind that the second sublot begins at station 499 + 35 (sta. 486 + 15 + 1320'), the second, third and fourth locations are determined by the same technique. These values are:

Sublot # 2 Sta. 509 + 42, 2.5' from base line.
Sublot # 3 Sta. 513 + 12, 9.2' from base line.
Sublot # 4 Sta. 536 + 44, 9.3' from base line.

SI EXAMPLE:

Stratification into sublots is accomplished by dividing the "Lot" material (in this case, 2,000 meters) into "four sublots" [each of 500 meters (2,000/4) long].

To locate a sample point station in sublot No. 1, the length of that sublot is multiplied by the "X" coordinate for the "sublot" and the product added to the beginning station for that sublot.

Starting Station = 1 + 525   1 + 525
$(X_1)(500) = (0.338) (500) = 169$ meters   169
Sample Station =   1 + 694

The sample point distance from the base line (generally centerline or the edge of pavement) is determined by multiplying Y_1 by the available width.

$(Y_1)(3.7$ m$) = (0.922) (3.7$ m$) = 3.4$ m from base line.

Thus the sample location is Sta. 1 + 694, 3.4 m from base line. Keeping in mind that the second sublot begins at station 2 + 025 (sta. 1 + 525 + 500 m), the second, third and fourth locations are determined by the same technique. These values are:

Sublot # 2 Sta. 2 + 407, 0.7 m from base line.
Sublot # 3 Sta. 2 + 547, 2.7 m from base line.
Sublot # 4 Sta. 3 + 430, 2.8 m from base line.

4. Methods for selection of random numbers.

4.1. Use of calculators or spreadsheet functions.

4.1.1. Many calculators have a random function. Review the manual for a given calculator to determine how to access this function. Sets of random numbers may be generated directly from the calculator by repeated use of this function.

4.1.2. Most spreadsheets also have a function to generate random numbers. Insert the random number function into a cell and press enter. A random number will be generated. Copy that cell as needed to produce the required quantity of random numbers. It may be necessary to reformat the cells to have only three decimal places. Read the manual for the specific spreadsheet for more detail on use.

NOTE: The District Materials Engineer may require a different method of generating random numbers to be used if an electronic method is determined to not be truly random.

4.2. Use of the Random Number Table (**Table 1**).

4.2.1. Use of the random number table requires the use of "seed" numbers to provide starting points for selection of the random numbers. A seed number can be obtained by several methods including odometer readings, generation by a random number function of a calculator or spreadsheet or by "pointing" if necessary.

4.2.1.1. Using an odometer reading such as 37864.2 as a seed number, use the digit farthest to the right (2) to select the column in the table. Use the two digits to the left of the decimal point, (64) to select the

row. If the seed number for the column is 0, use column 10 and if the seed for the row is 00, use row 100. In this case finding the intersection of the row and the column yields the number .338. Use this as a starting position and count down the column for the required number of samples. Selecting numbers for an X coordinate for three samples yields 0.338, 0.763 and 0.043. If a Y coordinate is also required use the fourth digit from the right for the column and the next two digits to the left, for the row. In this example that would yield column 8 and row 37 producing a starting point at number 0.922. If a total of three samples are required, counting down two more places yields numbers 0.198 and 0.737. Using this example, pairs of numbers for determining three X and Y coordinates are obtained, (0.338, 0.922), (0.763, 0.198) and (0.043, 0.737). Any amount of numbers required may be selected this way. If ten samples are required count down the column until ten numbers are selected. Once the bottom of a column has been reached go to the top of the next column to the right and countdown to obtain more numbers, if the bottom of column 10 is reached go to the top of column 1. Use of the odometer to generate seed numbers is not recommended if more than one set of X and Y pairs of random numbers is required in a relatively short period of time due to the slow change of the left odometer numbers.

4.2.1.2. Seed numbers may be obtained by using the random number function of a calculator or spreadsheet. In the above example the same results would have occurred if a calculator returned .264 for the first seed number. Use the first digit (2) to select a column and the second two digits (64) for the row. If using the random number function again produced 0.837, and then the same numbers would have been generated for the Y coordinate as in the above example.

4.2.1.3. Seed numbers may be obtained by "pointing" also. Lay copies of both pages of **Table 1** side by side and with eyes closed place a pointer on the table to select a seed number. Use this number as in the above example. Suitable pointers would be any device with a small tip such as a pin or a mechanical pencil.

Table 1 Random Numbers

	1	2	3	4	5	6	7	8	9	10
1	0.293	0.971	0.892	0.865	0.500	0.652	0.058	0.119	0.403	0.234
2	0.607	0.840	0.428	0.857	0.125	0.143	0.562	0.692	0.743	0.306
3	0.161	0.182	0.544	0.646	0.548	0.384	0.347	0.330	0.869	0.958
4	0.856	0.103	0.019	0.990	0.370	0.094	0.967	0.642	0.332	0.717
5	0.779	0.795	0.262	0.276	0.236	0.537	0.465	0.712	0.358	0.090
6	0.036	0.475	0.100	0.813	0.191	0.581	0.350	0.429	0.768	0.574
7	0.028	0.569	0.915	0.344	0.009	0.523	0.520	0.521	0.002	0.970
8	0.442	0.320	0.084	0.623	0.859	0.608	0.714	0.937	0.559	0.943
9	0.045	0.878	0.108	0.876	0.466	0.117	0.005	0.912	0.150	0.887
10	0.625	0.906	0.957	0.145	0.616	0.606	0.279	0.207	0.337	0.242
11	0.962	0.457	0.424	0.102	0.462	0.885	0.710	0.352	0.617	0.781
12	0.938	0.696	0.085	0.916	0.844	0.281	0.254	0.528	0.470	0.267
13	0.431	0.960	0.653	0.256	0.944	0.928	0.809	0.543	0.739	0.776
14	0.755	1.000	0.072	0.501	0.805	0.884	0.322	0.235	0.348	0.900
15	0.139	0.365	0.993	0.091	0.599	0.954	0.693	0.249	0.925	0.637
16	0.064	0.040	0.219	0.199	0.055	0.732	0.105	0.505	0.661	0.579
17	0.701	0.450	0.950	0.218	0.067	0.531	0.979	0.783	0.934	0.096
18	0.659	0.406	0.800	0.525	0.339	0.936	0.719	0.029	0.825	0.215
19	0.804	0.580	0.754	0.690	0.629	0.794	0.841	0.131	0.388	0.168
20	0.261	0.456	0.158	0.774	0.673	0.289	0.982	0.371	0.666	0.121
21	0.604	0.471	0.020	0.870	0.624	0.349	0.426	0.529	0.634	0.214
22	0.587	0.083	0.635	0.038	0.767	0.473	0.939	0.647	0.449	0.691
23	0.947	0.292	0.217	0.183	0.366	0.172	0.156	0.570	0.583	0.185
24	0.351	0.025	0.224	0.432	0.752	0.636	0.664	0.582	0.622	0.213
25	0.165	0.184	0.516	0.099	0.353	0.920	0.097	0.519	0.197	0.126
26	0.725	0.931	0.309	0.436	0.782	0.389	0.707	0.297	0.709	0.803
27	0.253	0.506	0.656	0.343	0.974	0.898	0.162	0.879	0.393	0.231
28	0.498	0.414	0.576	0.427	0.662	0.345	0.877	0.385	0.122	0.051
29	0.104	0.301	0.346	0.905	0.918	0.572	0.838	0.092	0.282	0.260
30	0.035	0.075	0.518	0.280	0.115	0.611	0.362	0.062	0.578	0.567
31	0.503	0.421	0.697	0.610	0.147	0.049	0.545	0.452	0.852	0.497
32	0.274	0.205	0.778	0.472	0.245	0.951	0.671	0.932	0.713	0.731
33	0.314	0.032	0.468	0.493	0.252	0.833	0.812	0.445	0.904	0.324
34	0.400	0.422	0.592	0.854	0.832	0.527	0.605	0.797	0.089	0.455
35	0.807	0.593	0.989	0.997	0.910	0.722	0.645	0.534	0.021	0.327
36	0.118	0.377	0.711	0.871	0.024	0.251	0.433	0.814	0.577	0.216
37	0.007	0.288	0.372	0.727	0.014	0.259	0.037	0.922	0.460	0.230
38	0.476	0.011	0.265	0.188	0.317	0.603	0.981	0.198	0.853	0.977
39	0.275	0.700	0.745	0.535	0.179	0.902	0.706	0.737	0.133	0.748
40	0.721	0.237	0.283	0.070	0.644	0.614	0.942	0.747	0.123	0.880
41	0.980	0.716	0.819	0.079	0.526	0.071	0.828	0.536	0.463	0.909
42	0.359	0.789	0.135	0.555	0.394	0.444	0.775	0.269	0.510	0.845
43	0.733	0.598	0.059	0.921	0.816	0.381	0.454	0.477	0.596	0.250
44	0.192	0.968	0.430	0.699	0.295	0.383	0.266	0.401	0.542	0.286
45	0.354	0.799	0.004	0.232	0.633	0.682	0.638	0.897	0.485	0.695
46	0.496	0.012	0.243	0.985	0.355	0.612	0.315	0.760	0.392	0.541
47	0.494	0.113	0.773	0.867	0.824	0.976	0.323	0.134	0.761	0.911
48	0.780	0.687	0.318	0.202	0.331	0.264	0.670	0.848	0.114	0.495
49	0.023	0.027	0.930	0.031	0.843	0.730	0.919	0.858	0.866	0.360
50	0.086	0.335	0.631	0.247	0.120	0.965	0.675	0.999	0.601	0.948

Table 1 (Cont)

	1	2	3	4	5	6	7	8	9	10
51	0.940	0.312	0.994	0.564	0.946	0.886	0.016	0.112	0.169	0.241
52	0.547	0.336	0.382	0.017	0.836	0.632	0.175	0.053	0.441	0.821
53	0.376	0.620	0.399	0.765	0.618	0.203	0.530	0.124	0.132	0.326
54	0.586	0.268	0.109	0.378	0.434	0.734	0.551	0.894	0.464	0.321
55	0.018	0.409	0.539	0.144	0.703	0.180	0.478	0.688	0.929	0.674
56	0.588	0.227	0.896	0.758	0.826	0.504	0.512	0.026	0.863	0.481
57	0.305	0.689	0.137	0.319	0.558	0.418	0.277	0.992	0.766	0.447
58	0.831	0.899	0.208	0.698	0.676	0.195	0.808	0.759	0.738	0.439
59	0.626	0.827	0.959	0.440	0.411	0.861	0.850	0.686	0.159	0.374
60	0.201	0.895	0.480	0.270	0.369	0.407	0.082	0.749	0.057	0.435
61	0.030	0.167	0.509	0.419	0.508	0.181	0.490	0.875	0.830	0.482
62	0.136	0.065	0.416	0.116	0.907	0.556	0.095	0.110	0.395	0.736
63	0.591	0.600	0.405	0.657	0.013	0.651	0.225	0.340	0.146	0.155
64	0.487	0.338	0.170	0.006	0.263	0.173	0.228	0.008	0.010	0.313
65	0.364	0.763	0.391	0.790	0.589	0.003	0.998	0.257	0.984	0.437
66	0.996	0.043	0.793	0.522	0.705	0.248	0.924	0.609	0.639	0.423
67	0.063	0.810	0.189	0.769	0.488	0.152	0.221	0.978	0.329	0.229
68	0.513	0.333	0.540	0.160	0.461	0.683	0.285	0.750	0.557	0.311
69	0.176	0.054	0.341	0.484	0.860	0.046	0.278	0.244	0.222	0.864
70	0.549	0.835	0.398	0.829	0.459	0.153	0.728	0.822	0.106	0.756
71	0.298	0.514	0.945	0.648	0.784	0.154	0.499	0.415	0.397	0.255
72	0.888	0.764	0.602	0.220	0.684	0.081	0.868	0.272	0.987	0.802
73	0.654	0.995	0.073	0.655	0.041	0.811	0.367	0.226	0.438	0.107
74	0.650	0.467	0.210	0.204	0.762	0.420	0.680	0.334	0.723	0.446
75	0.039	0.022	0.823	0.087	0.076	0.568	0.515	0.223	0.561	0.316
76	0.291	0.791	0.788	0.396	0.212	0.138	0.357	0.304	0.575	0.342
77	0.834	0.373	0.584	0.694	0.613	0.817	0.129	0.546	0.425	0.290
78	0.511	0.375	0.048	0.923	0.001	0.088	0.258	0.166	0.787	0.837
79	0.538	0.174	0.068	0.052	0.640	0.148	0.093	0.553	0.565	0.862
80	0.560	0.724	0.975	0.818	0.796	0.379	0.069	0.034	0.792	0.757
81	0.492	0.820	0.489	0.872	0.770	0.991	0.704	0.050	0.874	0.621
82	0.890	0.356	0.451	0.554	0.649	0.507	0.061	0.479	0.211	0.273
83	0.966	0.798	0.917	0.141	0.368	0.193	0.443	0.751	0.458	0.746
84	0.517	0.715	0.777	0.742	0.839	0.307	0.246	0.956	0.665	0.111
85	0.786	0.328	0.015	0.643	0.882	0.815	0.963	0.590	0.855	0.891
86	0.047	0.702	0.287	0.177	0.164	0.552	0.296	0.413	0.941	0.849
87	0.681	0.678	0.563	0.851	0.726	0.801	0.573	0.056	0.140	0.641
88	0.404	0.842	0.412	0.893	0.935	0.744	0.386	0.299	0.178	0.881
89	0.033	0.042	0.753	0.660	0.685	0.171	0.408	0.060	0.550	0.302
90	0.128	0.658	0.667	0.926	0.239	0.127	0.903	0.483	0.300	0.597
91	0.973	0.933	0.361	0.595	0.186	0.901	0.914	0.190	0.303	0.098
92	0.672	0.729	0.163	0.310	0.196	0.964	0.486	0.308	0.735	0.474
93	0.524	0.402	0.628	0.410	0.846	0.206	0.585	0.566	0.044	0.627
94	0.720	0.157	0.238	0.078	0.233	0.771	0.533	0.986	0.077	0.101
95	0.983	0.669	0.927	0.066	0.080	0.740	0.969	0.630	0.619	0.200
96	0.294	0.387	0.988	0.961	0.913	0.679	0.284	0.949	0.380	0.785
97	0.668	0.149	0.972	0.187	0.151	0.502	0.718	0.453	0.953	0.491
98	0.130	0.708	0.417	0.594	0.209	0.663	0.908	0.271	0.532	0.741
99	0.883	0.677	0.615	0.469	0.363	0.142	0.952	0.325	0.194	0.847
100	0.889	0.772	0.390	0.571	0.873	0.806	0.448	0.955	0.240	0.074

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5.2.6 COMPARISON OF QUALITY CONTROL AND VERIFICATION TESTS

This procedure is carried out to compare two different sets of multiple test results for finding the same parameter. Typical example would be comparing contractor QC test results and KDOT verification test results to determine if the material under test came from the same population. The statistical test which would be used to compare two means would be popularly known as Student's t-test or simply t-tests for testing a null hypothesis (H_0) with certain confidence (e.g. 99%) or level of significance (risk of rejecting a null hypothesis when it is true, e.g., 1%) is as follows:

H_o :   There is no difference in the sample means, ie. the means are statistically equal

If the test results do not support this hypothesis than an alternate hypothesis (H_a) is accepted as:

H_a: The means are different, ie. the means are not statistically equal

This test is generally applicable when the number of tests (or observations as is known in Statistics) is less than or equal to 30. However, since the approach used in the t-test is dependent upon whether or not the variances (square of the sample standard deviation) are equal for the two sets of data, it is necessary to test the variances of the test results before comparing the means of the test results.

F-test for the Sample Variances

The F-test determines whether the difference in the variability of the contractor's QC tests and that of KDOT's verification tests is larger than might be expected from chance if they came from the same population. In this case, a hypothesis testing is done at a certain level of significance. The null hypothesis in the test is:

H_o: There is no difference in the sample variances, ie. the variances are statistically equal

If the test results do not support this hypothesis than an alternate hypothesis is accepted as:

H_a: The variances are different, i.e. the variances are not statistically equal

The following steps need to be followed in doing an F-test:

i)   Compute the variance (the standard deviation squared) for the QC tests, s_c^2, and the KDOT verification tests, s_v^2

ii)   Compute F statistic as:

$$F = s_c^2 / s_v^2 \text{ or } s_v^2 / s_c^2$$

Always use the larger of the two variances in the numerator.

iii)   Choose the level of significance, α, for the test. The recommended α is 1%.

iv)   Find the critical F value F_{crit}, from the **Table 1** using the degrees of freedom associated with each set of test results. The degrees of freedom for each set of results is the number of test results in the set, less one. If the number of QC tests *is n_c* and the number of verification tests is n_v, then the degrees of freedom associated with s_c^2 is *(n_c-1)* and the degrees of freedom associated with s_v^2 is *(n_v-1*). The values in Table 5.17.08-1 are tabulated to test if there is a difference (either larger or smaller) between two variance estimates. This is known as a two-sided or two-tailed test.   Care must be taken when using other tables of the F distribution , since they are usually based on a one-tailed test, ie., testing specifically whether one variance is larger than another. When finding F_{crit} be sure that the appropriate degrees of freedom for the numerator and denominator are used.

v)   Find the value for F_{crit} from **Table 1**.

vi)   If $F \geq F_{crit}$, then the null hypothesis is rejected i.e. the two sets of tests have significantly different variabilities. If $F < F_{crit}$ then there is no reason to believe that the variabilities are significantly different.

t-test for Sample Means

Once the variances have been tested and been assumed to be either equal or not equal, the means of the test results can be tested to determine whether they differ from one another or can be assumed equal. The desire is to determine whether it is reasonable to assume that the QC tests came from the same population as the verification tests. As mentioned before, a t-test is used to compare the sample means. Two approaches for the t-test are necessary.

If the sample variances are assumed equal, then the t-test is conducted based on the two samples using a *pooled estimate for the variance (s_p^2)* and the pooled degrees of freedom. If the sample variances are found to be different in the F-test, the t-test is conducted using the individual sample variances, the individual sample sizes, and the effective degrees of freedom (estimated from the sample variances and sample sizes).

In either of the two cases discussed earlier, the null hypothesis used is:

> *H_o :   There is no difference in the sample means, i.e. the means are statistically*
> *   equal*

If the test results do not support this hypothesis than an alternate hypothesis is accepted as:
> *H_a: The means are different, i.e. the means are not statistically equal*

Table 1   Critical Values, F_{crit} for the F-test for a Level of Significance, $\alpha = 1\%$

DEGREES OF FREEDOM FOR NUMERATOR

	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

DEGREES OF FREEDOM FOR DENOMINATOR

NOTE : This is for a *two-tailed test* with the null and alternate hypotheses shown below:

$$H_o : \quad s^2_c = s^2_v$$
$$H_a : \quad s^2_c \neq s^2_v$$

Table 1 **Critical Values, F$_{crit}$, for the F-test for a Level of Significance, α =1 % (contd..)**

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	15	20	24	30	40	50	60	100	120	200	500	∞
1	24600	24800	24900	25000	25100	25200	25300	25300	25400	25400	25400	25500
2	199	199	199	199	199	199	199	199	199	199	199	200
3	43.1	42.8	42.69	42.5	42.3	42.2	42.1	42.0	42.0	41.9	41.9	41.8
4	20.4	20.2	20.0	19.9	19.8	19.7	19.6	19.5	19.5	19.4	19.4	19.3
5	13.1	12.9	12.8	12.7	12.5	12.5	12.4	12.3	12.3	12.2	12.2	12.1
6	9.81	9.59	9.47	9.36	9.24	9.17	9.12	9.03	9.00	8.95	8.91	8.88
7	7.97	7.75	7.65	7.53	7.42	7.35	7.31	7.22	7.19	7.15	7.10	7.08
8	6.81	6.61	6.50	6.40	6.29	6.22	6.18	6.09	6.06	6.02	5.98	5.95
9	6.03	5.83	5.73	5.62	5.52	5.45	5.41	5.32	5.30	5.26	5.21	5.19
10	5.47	5.27	5.17	5.07	4.97	4.90	4.86	4.77	4.75	4.71	4.67	4.64
11	5.05	4.86	4.76	4.65	4.55	4.49	4.45	4.36	4.34	4.29	4.25	4.23
12	4.72	4.53	4.43	4.33	4.23	4.17	4.12	4.04	4.01	3.97	3.93	3.90
15	4.07	3.88	3.79	3.69	3.59	3.52	3.48	3.39	3.37	3.33	3.29	3.26
20	3.50	3.32	3.22	3.12	3.02	2.96	2.92	2.83	2.81	2.76	2.72	2.69
24	3.25	3.06	2.97	2.87	2.77	2.70	2.66	2.57	2.55	2.50	2.46	2.43
30	3.01	2.82	2.73	2.63	2.52	2.46	2.42	2.32	2.30	2.25	2.21	2.18
40	2.78	2.60	2.50	2.40	2.3	2.23	2.18	2.09	2.06	2.01	1.96	1.93
60	2.57	2.39	2.29	2.19	2.08	2.01	1.96	1.86	1.83	1.78	1.73	1.69
120	2.37	2.19	2.09	1.98	1.87	1.80	1.75	1.64	1.61	1.54	1.48	1.43
∞	2.19	2.00	1.90	1.79	1.67	1.59	1.53	1.40	1.36	1.28	1.17	1.00

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

$$H_o : \quad s^2_c = s^2_v$$
$$H_a : \quad s^2_c \neq s^2_v$$

5.2.6   2016

Revised 2016

Case 1:   **Sample Variances Assumed to Be Equal**

    a)   To conduct the t-test when the sample variances are assumed equal, Equation 1 is used to calculate the t value from which the decision is reached.

$$t = \frac{\left| \overline{X}_c - \overline{X}_v \right|}{\sqrt{\dfrac{s_p^2}{n_c} + \dfrac{s_p^2}{n_v}}} \qquad\qquad (1)$$

where: $\overline{X}_c$ =   mean of QC tests

                      $\overline{X}_v$ =   mean of verification tests

                      s_p^2 =   pooled estimate for the variance (described below)

                      n_c =   number of QC tests

                      n_v =   number of verification tests

    b)   The pooled variance, which is the weighted average, using the degrees of freedom for each sample as the weighting factor, is computed from the sample variances using Equation 2.

$$s_p^2 = \frac{s_c^2\left(n_c - 1\right) + s_v^2\left(n_v - 1\right)}{n_c + n_v - 2} \qquad\qquad (2)$$

Where: s_p^2 =   pooled estimate for the variance

                      n_c =   number of QC tests

                      n_v =   number of verification tests

                      s_c^2 =   variance of the QC tests

                      s_v^2 =   variance of the verification tests

    c)   Once the pooled variance is estimated, the value of t is computed using equation 1.

    d)   To determine the critical t value against which to compare the computed t value, it is necessary to select the level of significance, α. *A value of $\alpha = 1$ % is recommended.*

    e)   Determine the critical t value, t_{crit}, from **Table 2** for the pooled degrees of freedom. The pooled degrees of freedom for the case where the sample variances are assumed equal is $(n_c + n_v - 2)$.

    f)   If $t \geq t_{crit}$, then decide that the two sets of tests have significantly different means. If $t < t_{crit}$, then decide that there is no reason to believe that the means are significantly different.

Case 2:   **Sample Variances Assumed to Be Not Equal**

a) To conduct the t-test when the sample variances are assumed not equal, Equation 3 is used to calculate the t value from which the decision is reached.

$$t = \frac{\left| \overline{X}_c - \overline{X}_v \right|}{\sqrt{\dfrac{s_C^2}{n_C} + \dfrac{s_v^2}{n_v}}} \qquad (3)$$

where:
$\overline{X}_c$ = mean of QC tests

$\overline{X}_v$ = mean of verification tests

s_c^2 = variance of the QC tests

s_v^2 = variance of the verification tests

n_c = number of QC tests

n_v = number of verification tests

b) To determine the critical t value against which to compare the computed t value, it is necessary to select the level of significance, α. A value of α = 1% is recommended.

c) The effective degrees of freedom, f', for the case where the sample variances are assumed not equal is determined from Equation 4 *(the calculated effective degrees of freedom is rounded down to a whole number).*

$$f' = \frac{\left(\dfrac{s_c^2}{n_c} + \dfrac{s_v^2}{n_v} \right)^2}{\left[\dfrac{\left(\dfrac{s_c^2}{n_c} \right)^2}{n_c + 1} + \dfrac{\left(\dfrac{s_v^2}{n_v} \right)^2}{n_v + 1} \right]} - 2 \qquad (4)$$

where all the symbols are as described previously.

d) Determine the critical t value, t_{crit}, from **Table 2** for the effective degrees of freedom determined by Equation 4.

e) If $t \geq t_{crit}$, then decide that the two sets of tests have significantly different means. If $t < t_{crit}$, then decide that there is no reason to believe that the means are significantly different.

5.2.6 COMPARISON OF QUALITY CONTROL AND VERIFICATION TESTS

This procedure is carried out to compare two different sets of multiple test results for finding the same parameter. Typical example would be comparing contractor QC test results and KDOT verification test results to determine if the material under test came from the same population. The statistical test which would be used to compare two means would be popularly known as Student's t-test or simply t-tests for testing a null hypothesis (H_0) with certain confidence (e.g. 99%) or level of significance (risk of rejecting a null hypothesis when it is true, e.g., 1%) is as follows:

H_o :   There is no difference in the sample means, ie. the means are statistically equal

If the test results do not support this hypothesis than an alternate hypothesis (H_a) is accepted as:

H_a: The means are different, ie. the means are not statistically equal

This test is generally applicable when the number of tests (or observations as is known in Statistics) is less than or equal to 30. However, since the approach used in the t-test is dependent upon whether or not the variances (square of the sample standard deviation) are equal for the two sets of data, it is necessary to test the variances of the test results before comparing the means of the test results.

F-test for the Sample Variances

The F-test determines whether the difference in the variability of the contractor's QC tests and that of KDOT's verification tests is larger than might be expected from chance if they came from the same population. In this case, a hypothesis testing is done at a certain level of significance. The null hypothesis in the test is:

H_o: There is no difference in the sample variances, ie. the variances are statistically equal

If the test results do not support this hypothesis than an alternate hypothesis is accepted as:

H_a: The variances are different, i.e. the variances are not statistically equal

The following steps need to be followed in doing an F-test:

i)   Compute the variance (the standard deviation squared) for the QC tests, s_c^2, and the KDOT verification tests, s_v^2

ii)   Compute F statistic as:

$$F = s_c^2 / s_v^2 \text{ or } s_v^2 / s_c^2$$

Always use the larger of the two variances in the numerator.

iii)   Choose the level of significance, α, for the test. The recommended α is 1%.

iv)   Find the critical F value F_{crit}, from the **Table 1** using the degrees of freedom associated with each set of test results. The degrees of freedom for each set of results is the number of test results in the set, less one. If the number of QC tests *is n_c* and the number of verification tests is n_v, then the degrees of freedom associated with s_c^2 is *(n_c-1)* and the degrees of freedom associated with s_v^2 is *(n_v-1)*. The values in Table 5.17.08-1 are tabulated to test if there is a difference (either larger or smaller) between two variance estimates. This is known as a two-sided or two-tailed test.   Care must be taken when using other tables of the F distribution , since they are usually based on a one-tailed test, ie., testing specifically whether one variance is larger than another. When finding F_{crit} be sure that the appropriate degrees of freedom for the numerator and denominator are used.

v)   Find the value for F_{crit} from **Table 1**.

vi)   If F ≥ F_{crit} , then the null hypothesis is rejected i.e. the two sets of tests have significantly different variabilities. If F < F_{crit} then there is no reason to believe that the variabilities are significantly different.

t-test for Sample Means

Once the variances have been tested and been assumed to be either equal or not equal, the means of the test results can be tested to determine whether they differ from one another or can be assumed equal. The desire is to determine whether it is reasonable to assume that the QC tests came from the same population as the verification tests. As mentioned before, a t-test is used to compare the sample means. Two approaches for the t-test are necessary.

If the sample variances are assumed equal, then the t-test is conducted based on the two samples using a *pooled estimate for the variance (s_p^2)* and the pooled degrees of freedom. If the sample variances are found to be different in the F-test, the t-test is conducted using the individual sample variances, the individual sample sizes, and the effective degrees of freedom (estimated from the sample variances and sample sizes).

In either of the two cases discussed earlier, the null hypothesis used is:

H_o :   *There is no difference in the sample means, i.e. the means are statistically*
          equal

If the test results do not support this hypothesis than an alternate hypothesis is accepted as:
          H_a: *The means are different, i.e. the means are not statistically equal*

Table 1   Critical Values, F_{crit} for the F-test for a Level of Significance, $\alpha = 1\%$

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

NOTE : This is for a *two-tailed test* with the null and alternate hypotheses shown below:

$$H_o : \quad s^2{}_c = s^2{}_v$$
$$H_a : \quad s^2{}_c \neq s^2{}_v$$

Revised 2016

Table 1   Critical Values, F_{crit} , for the F-test for a Level of Significance, α =1 % (contd..)

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR

	15	20	24	30	40	50	60	100	120	200	500	∞
1	24600	24800	24900	25000	25100	25200	25300	25300	25400	25400	25400	25500
2	199	199	199	199	199	199	199	199	199	199	199	200
3	43.1	42.8	42.69	42.5	42.3	42.2	42.1	42.0	42.0	41.9	41.9	41.8
4	20.4	20.2	20.0	19.9	19.8	19.7	19.6	19.5	19.5	19.4	19.4	19.3
5	13.1	12.9	12.8	12.7	12.5	12.5	12.4	12.3	12.3	12.2	12.2	12.1
6	9.81	9.59	9.47	9.36	9.24	9.17	9.12	9.03	9.00	8.95	8.91	8.88
7	7.97	7.75	7.65	7.53	7.42	7.35	7.31	7.22	7.19	7.15	7.10	7.08
8	6.81	6.61	6.50	6.40	6.29	6.22	6.18	6.09	6.06	6.02	5.98	5.95
9	6.03	5.83	5.73	5.62	5.52	5.45	5.41	5.32	5.30	5.26	5.21	5.19
10	5.47	5.27	5.17	5.07	4.97	4.90	4.86	4.77	4.75	4.71	4.67	4.64
11	5.05	4.86	4.76	4.65	4.55	4.49	4.45	4.36	4.34	4.29	4.25	4.23
12	4.72	4.53	4.43	4.33	4.23	4.17	4.12	4.04	4.01	3.97	3.93	3.90
15	4.07	3.88	3.79	3.69	3.59	3.52	3.48	3.39	3.37	3.33	3.29	3.26
20	3.50	3.32	3.22	3.12	3.02	2.96	2.92	2.83	2.81	2.76	2.72	2.69
24	3.25	3.06	2.97	2.87	2.77	2.70	2.66	2.57	2.55	2.50	2.46	2.43
30	3.01	2.82	2.73	2.63	2.52	2.46	2.42	2.32	2.30	2.25	2.21	2.18
40	2.78	2.60	2.50	2.40	2.3	2.23	2.18	2.09	2.06	2.01	1.96	1.93
60	2.57	2.39	2.29	2.19	2.08	2.01	1.96	1.86	1.83	1.78	1.73	1.69
120	2.37	2.19	2.09	1.98	1.87	1.80	1.75	1.64	1.61	1.54	1.48	1.43
∞	2.19	2.00	1.90	1.79	1.67	1.59	1.53	1.40	1.36	1.28	1.17	1.00

NOTE : This is for a ***two-tailed test*** with the null and alternate hypotheses shown below:

$$H_o : \quad s^2_c = s^2_v$$
$$H_a : \quad s^2_c \neq s^2_v$$

Revised 2016

Case 1:   **Sample Variances Assumed to Be Equal**

    a)   To conduct the t-test when the sample variances are assumed equal, Equation 1 is used to calculate the t value from which the decision is reached.

$$t = \frac{\left|\overline{X}_c - \overline{X}_v\right|}{\sqrt{\dfrac{s_p^2}{n_c} + \dfrac{s_p^2}{n_v}}} \qquad (1)$$

    where: $\overline{X}_c$ =   mean of QC tests

                         $\overline{X}_v$ =   mean of verification tests

                         s_p^2 =   pooled estimate for the variance (described below)

                         n_c =   number of QC tests

                         n_v =   number of verification tests

    b)   The pooled variance, which is the weighted average, using the degrees of freedom for each sample as the weighting factor, is computed from the sample variances using Equation 2.

$$s_p^2 = \frac{s_c^2\left(n_c - 1\right) + s_v^2\left(n_v - 1\right)}{n_c + n_v - 2} \qquad (2)$$

    Where: s_p^2 =   pooled estimate for the variance

                         n_c =   number of QC tests

                         n_v =   number of verification tests

                         s_c^2 =   variance of the QC tests

                         s_v^2 =   variance of the verification tests

    c)   Once the pooled variance is estimated, the value of t is computed using equation 1.

    d)   To determine the critical t value against which to compare the computed t value, it is necessary to select the level of significance, α. *A value of $\alpha = 1\%$ is recommended.*

    e)   Determine the critical t value, t_{crit}, from **Table 2** for the pooled degrees of freedom. The pooled degrees of freedom for the case where the sample variances are assumed equal is $(n_c + n_v - 2)$.

    f)   If $t \geq t_{crit}$, then decide that the two sets of tests have significantly different means. If $t < t_{crit}$, then decide that there is no reason to believe that the means are significantly different.

Case 2:   Sample Variances Assumed to Be Not Equal

a)   To conduct the t-test when the sample variances are assumed not equal, Equation 3 is used to calculate the t value from which the decision is reached.

$$t = \frac{\left| \overline{X}_c - \overline{X}_v \right|}{\sqrt{\dfrac{s_c^2}{n_c} + \dfrac{s_v^2}{n_v}}} \qquad (3)$$

where: $\overline{X}_c$ =   mean of QC tests

$\overline{X}_v$ =   mean of verification tests

s_c^2 =   variance of the QC tests

s_v^2 =   variance of the verification tests

n_c =   number of QC tests

n_v =   number of verification tests

b)   To determine the critical t value against which to compare the computed t value, it is necessary to select the level of significance, α. A value of $\alpha = 1\%$ is recommended.

c)   The effective degrees of freedom, f', for the case where the sample variances are assumed not equal is determined from Equation 4 *(the calculated effective degrees of freedom is rounded down to a whole number)*.

$$f' = \frac{\left(\dfrac{s_c^2}{n_c} + \dfrac{s_v^2}{n_v} \right)^2}{\left(\dfrac{\left(\dfrac{s_c^2}{n_c} \right)^2}{n_c + 1} + \dfrac{\left(\dfrac{s_v^2}{n_v} \right)^2}{n_v + 1} \right)} - 2 \qquad (4)$$

where all the symbols are as described previously.

d)   Determine the critical t value, t_{crit}, from **Table 2** for the effective degrees of freedom determined by Equation 4.

e)   If $t \geq t_{crit}$, then decide that the two sets of tests have significantly different means. If $t < t_{crit}$, then decide that there is no reason to believe that the means are significantly different.

Table 2   **Critical t values**

degrees of freedom	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

NOTE : This is for a two-tailed test with the null and alternate hypotheses shown below :

$$H_o : \quad \overline{X}_c = \overline{X}_v$$

$$H_a : \quad \overline{X}_c \neq \overline{X}_v$$

Example Problem 1-Concrete

A contractor has run 21 QC tests for compressive strength and KDOT has run 5 verification tests over the same period of time. The results are shown below. Is it likely that the tests came from the same population?

Contractor QC Test Results
(%)
36.40
36.65
32.69
38.05
38.54
37.59
36.57
42.48
36.99
38.20
37.53
36.00
41.28
40.00
38.37
38.72
40.36
30.37
34.87
35.62
36.06
$\overline{X}_c = 37.302$

KDOT VerificationTest Results
(%)
36.10
30.00
37.00
32.80
30.60
$\overline{X}_v = 33.300$

A t-test between the means of these two sets of results can be used to test whether the mean results of the tests done by the contractor and KDOT are statistically different. If they are not different, then it is likely that they came from the same population. However, first the F-test needs to be done to determine whether or not to assume the variance of the QC test results differs from the KDOT verification tests.

      Step 1.   Compute the mean and standard deviation for each set of data:

      <u>QC test results</u>   <u>KDOT Verification test results</u>

      $\overline{X}_c = 37.302$ $\overline{X}_v = 33.300$

      $s_c = 2.736$ $s_v = 3.161$

      Step 2.   Compute variance, s^2, for each set of test results (variance is square of the standard deviation):

| QC test results | KDOT Verification test results |

$$s_c^2 = 7.431 \qquad\qquad\qquad s_v^2 = 9.992$$

Step 3. Compute F, **using the largest s^2 in the numerator**.

$$F = \frac{s_v^2}{s_c^2} = \frac{9.992}{7.431} = 1.34$$

Step 4. Determine F_{crit} from **Table 1** being sure to use the correct degrees of freedom for the numerator ($n_v - 1 = 5 - 1 = 4$) and the denominator ($n_c - 1 = 21 - 1 = 20$).

From Table 5.17.08-1, at $\alpha = 1\%$, $\qquad\qquad\qquad$ $F_{crit} = 5.17$

Conclusion: Since $F < F_{crit}$ (i.e., $1.34 < 5.17$), there is no reason to believe that the two sets of tests have different variabilities. That is, they could have come from the same population. Since we can assume that the variances are equal, we can use *the pooled variance* to calculate the t-test statistic, and *the pooled degrees of freedom* to determine the critical t value, t_{crit}.

Step 5. Compute the pooled variance, s_p^2, using the sample variances from above.

$$s_p^2 = \frac{s_c^2(n_c - 1) + s_v^2(n_v - 1)}{n_c + n_v - 2}$$

$$s_p^2 = \frac{(7.431)(20) + (9.992)(4)}{21 + 5 - 2} = 7.86$$

Step 6. Compute the t-test statistic, t.

$$t = \frac{\left|\overline{X}_c - \overline{X}_v\right|}{\sqrt{\dfrac{s_p^2}{n_c} + \dfrac{s_p^2}{n_v}}}$$

$$t = \frac{\left|37.302 - 33.300\right|}{\sqrt{\dfrac{7.86}{21} + \dfrac{7.86}{5}}} = \frac{4.002}{\sqrt{1.946}} = 2.87$$

Step 7. Determine the critical t value, t_{crit}, for the pooled degrees of freedom degrees of freedom = $(n_c + n_v - 2) = (21 + 5 - 2) = 24$.

From **Table 2**,

for $\alpha = 1$ % and degrees of freedom = 24,

$t_{crit} = 2.80$.

Conclusion: Since $2.87 > 2.80$, we assume that the sample means are not equal. It is therefore probable that the two sets of test results did not come from the same population (or lot).

Example Problem - Case 2-Asphalt

A contractor has run 10 QC tests and KDOT has run 5 verification tests over the same period of time for the asphalt pavement density (%G_{mm}). The results are shown below. Is it likely that the test came from the same population or lot?

Contractor QC Test Results	KDOT Verification Test Results
93.0	95.5
92.4	93.3
92.9	94.1
93.6	92.5
92.9	92.7
92.9	$\overline{X}_v = 93.62$
92.4	
93.4	
92.9	
92.4	
$\overline{X}_c = 92.88$	

A t-test between the means of these two sets of results can be used to test whether the mean results of the %G_{mm} done by the contractor and KDOT are statistically different. If they are not different, then it is likely that they came from the same population. First, we have to determine whether the variance of the QC tests differ from the verification tests using F-test.

Step 1. Compute the mean and standard deviation for each set of data:

QC test results   KDOT Verification test results

$\overline{X}_c = 92.88$ $\overline{X}_v = 93.62$

$s_c = 0.408$ $s_v = 1.221$

Step 2. Compute the variance, s^2, for each set of tests (variance is the square of the standard deviation):

$s_c^2 = 0.166$ $s_v^2 = 1.491$

Step 3.   Compute F, **using the largest s^2 in the numerator**.

$$F = \frac{s_v^2}{s_c^2} = \frac{1.491}{0.166} = 8.98$$

Step 4.   Determine F_{crit} from **Table 1** (be sure to use the correct degrees of freedom for the numerator ($n_v - 1 = 5 - 1 = 4$) and the denominator ($n_c - 1 = 10 - 1 = 9$)).

From **Table 1**, at $\alpha = 1\ \%$, $F_{crit} = 7.96$

Conclusion: Since $F > F_{crit}$ (i.e., $8.98 > 7.96$), there is reason to believe that the two sets of tests have different variabilities.   Thus, it is likely that they came from populations with different variances.   Since we CAN NOT assume that the variances are equal, we cannot use the pooled variance to calculate the t-test statistic, and the pooled degrees of freedom to determine the critical t value, t_{crit}.

Step 5.   Compute the t-test statistic, t.

$$t = \frac{\left| \overline{X}_c - \overline{X}_v \right|}{\sqrt{\dfrac{s_c^2}{n_c} + \dfrac{s_v^2}{n_v}}}$$

$$t = \frac{\left| 92.88 - 93.62 \right|}{\sqrt{\dfrac{0.166}{10} + \dfrac{1.491}{5}}} = \frac{0.74}{\sqrt{0.315}} = 1.32$$

Step 6.   Determine the critical t value, t_{crit}, for the approximate degrees of freedom *(the calculated effective degrees of freedom is rounded down to a whole number)*.

$$f' = \frac{\left(\dfrac{s_c^2}{n_c} + \dfrac{s_v^2}{n_v} \right)^2}{\left(\dfrac{\left(\dfrac{s_c^2}{n_c} \right)^2}{n_c + 1} + \dfrac{\left(\dfrac{s_v^2}{n_v} \right)^2}{n_v + 1} \right)} - 2$$

$$f'=\frac{\left(\dfrac{0.166}{10}+\dfrac{1.491}{5}\right)^2}{\left(\dfrac{\left(\dfrac{0.166}{10}\right)^2}{11}+\dfrac{\left(\dfrac{1.491}{5}\right)^2}{6}\right)}-2=\frac{(0.315)^2}{0.0148}-2=4.7$$

From **Table 2**,
for $\alpha = 1\%$ and degrees of freedom = 4 (rounded down to the nearest whole number)

$t_{crit} = 4.60$

Conclusion: Since $t < t_{crit}$, (i.e., $1.32 < 4.60$), there is no reason to assume that the sample means are not equal. It is, therefore, reasonable to assume that the sets of test results came from populations that had the same mean.

Asphalt Paving Excel Spreadsheet

The Air Voids F & t portion of the EXCEL spreadsheet compares the Contractor's Quality Control (QC) results and KDOT's verification results using the following process:

In lots 1 and 2, the mean and standard deviation of the QC results are calculated and compared to the mean of the verification results. The comparison is considered to be satisfactory (Pass) if the mean of the verification results for that lot is within the greater of:

1.   the mean of the QC results ± three standard deviations of the QC results for that lot
2.   one percent of the mean of the QC results for that lot

Starting with lot 3, the F & t tests are used to compare the QC results and verification results. All of the QC results and verification results are used in the comparison for lots 3, 4 and 5. Starting with lot 6, all of the QC results and verification results for the last five lots are used in the comparison. For example, the test results from lots 2-6 are used in the comparison for lot 6.

The maximum specific gravity (G_{mm}) F & t portion of the EXCEL spreadsheet compares the QC results and verification results using the follow process:

In lots 1 and 2, the mean and standard deviation of the QC results are calculated and compared to the mean of the verification results. The comparison is considered to be satisfactory (Pass) if the mean of the verification results for that lot is within the greater of :

1.   the mean of the QC results ± three standard deviations of the QC results for that lot
2.   0.02 of the mean of the QC results for that lot

Starting with lot 3, the F & t tests are used to compare the QC results and verification results. All of the QC results and verification results are used in the comparison for lots 3, 4 and 5. Starting with lot 6, all of the QC results and verification results for the last five lots are used in the comparison. For example, the test results from lots 2-6 are using in the comparison for lot 6.

If the results of comparison of the G_{mm} QC and verification results for a lot are satisfactory (Pass), the QC G_{mm} results should be used in the calculation of $\%G_{mm}$ for both the QC and verification Density results. If the results of the comparison of the G_{mm} QC and verification results for a lot are not satisfactory (Fail), the verification G_{mm} results should be used in the calculation of $\%G_{mm}$ for both the QC and verification Density results.

The Density F & t portion of the EXCEL spreadsheet compares the QC results and verification results using the follow process:

All of a lot's QC results and verification results are used in the comparison for that lot. Each lot stands on its' own for the Density F & t comparison.

For the Air Voids, G_{mm} and Density F & t comparisons, the results are considered satisfactory (Pass) if the t-test shows that the Contractor's QC results and KDOT's QA results are from the same population with a α of 1%.

Concrete Paving Excel Spreadsheet

The Compressive Strength and Thickness F & t portions of the EXCEL spreadsheet compare the Contractor's Quality Control (QC) results and KDOT's verification results using the following process:

In lots 1 and 2, the mean and standard deviation of the QC results are calculated and compared to the mean of the verification results. The comparison is considered to be satisfactory (Pass) if the mean of the verification results for that lot is within the mean of the QC results ± three standard deviations of the QC results for that lot.

Starting with lot 3, the F & t tests are used to compare the QC results and verification results. All of the QC results and verification results are used in the comparison for lots 3, 4 and 5. Starting with lot 6, all of the QC results and verification results for the last five lots are used in the comparison. For example, the test results from lots 2-6 are used in the comparison for lot 6.

For the Compressive Strength and Thickness F & t comparisons, the results are considered satisfactory (Pass) if the t-test shows that the Contractor's QC results and KDOT's QA results are from the same population with a α of 1%.

F & t Air Void Spreadsheet

HMA 1R 602		07-06006-R1	SEM 03/01/2009								
Lots:	1	10	**Project #**	KA 1441-01		**Name of QC Tester**					
Dates:	3/25/2009	4/29/2009	**Contract #**	508106343		**Certification # of QC Tester**					
Mix Type		SR-12.5A	64-28	HMA Overlay		**Contract Line #'s**	08 12				
Metric/English:		E		**Placement**	ML						
				Air Voids							
Lot	**Date**	**Contractor Quality Control Tests (%)**	**KDOT Verification Test (%)**	**Number of Contractor Tests**	**Number of KDOT Tests**	**t Test**	**t(crit)**	**Are Means The Same?**	**Use Contractor Test Results?**	**Comments**	
1A	3/25/2009	3.81									
1B	3/31/2009	4.88									
1C	3/31/2009	4.41									
1D	4/2/2009	2.92	3.08								
1E											
1F				4	1			Pass	Yes		
2A	4/2/2009	3.97									
2B	4/3/2009	4.11									
2C	4/3/2009	4.29	4.65								
2D	4/3/2009	4.35									
2E											
2F				4	1			Pass	Yes		
3A	4/7/2009	4.48									
3B	4/7/2009	4.41	5.06								
3C	4/8/2009	4.30									
3D	4/8/2009	4.66									
3E											
3F				12	3	0.12	3.01	Pass	Yes		
4A	4/8/2009	4.85									
4B	4/9/2009	4.79	4.94								
4C	4/10/2009	4.41									
4D	4/10/2009	3.62									
4E											
4F				16	4	0.50	2.88	Pass	Yes		
5A	4/13/2009	4.32									
5B	4/13/2009	4.52	4.47								
5C	4/14/2009	3.51									
5D	4/14/2009	4.50									
5E											
5F				20	5	0.67	2.81	Pass	Yes		

F & t Density Spreadsheet

HMA 1R	602 07-06006-R1					
Lots:	1	22	**Project #**	KA 1441-01	**Name of QC Tester**	
Dates:	3/25/2009	4/30/2009	**Contract #**	508106343	**Certification # of QC Tester**	
Mix Type	SR-12.5A	64-28	**HMA Overlay**	**Contract Line # 's**		

Lot	Date	Contractor Test Results (lb/ft3)	Gmm	Contractor Quality Control Tests (%Gmm)	KDOT Test Results (lb/ft3)	KDOT Verification Tests (%Gmm)	Number of Contractor Tests	Number of KDOT Tests	t Test	t(crit)	Are Means The Same?	Use Contractor Test Results?	Comments
1A1		139.8		92.51	134.9	89.26							
1A2		141.6		93.70									
1B1		139		91.98	134.1	88.73							
1B2		140.5		92.97									
1C1	3/25/2009	138.3		91.51	138.5	91.65							
1C2		139		91.98									
1D1		139		91.98	135	89.33							
1D2		139.4		92.24									
1E1													
1E2			2.428				8	4	4.65	3.17	Fail	No	
2A1		140.5		92.47	142	93.46							
2A2		140.4		92.41									
2B1		140.2		92.28	137.7	90.63							
2B2		137.2		90.30									
2C1	3/31/2009	139.5		91.82	140.2	92.28							
2C2		141.8		93.33									
2D1		139.8		92.01	137.7	90.63							
2D2		140.4		92.41									
2E1													
2E2			2.441				8	4	0.59	3.17	Pass	Yes	
3A1		142.3		93.66	139.1	91.55							
3A2		140.3		92.34									
3B1		140		92.14	138.6	91.22							
3B2		140.4		92.41									
3C1	4/1/2009	139.6		91.88	144	94.78							
3C2		140.1		92.21									
3D1		141.4		93.07	140.7	92.61							
3D2		141.2		92.93									
3E1		139.6		91.88	139.7	91.95							
3E2		141	2.441	92.80			10	5	0.22	3.01	Pass	Yes	
4A1		142.3		93.51	139.9	91.93							
4A2		142		93.31									
4B1		141		92.65	140.6	92.39							
4B2		141.8		93.18									
4C1	4/2/2009	140.6		92.39	141	92.65							
4C2		140.2		92.13									
4D1		140.4		92.26	142.7	93.77							
4D2		140.3		92.19									
4E1		140.7		92.45	140.4	92.26							
4E2		139.9	2.445	91.93			10	5	0.00	3.01	Pass	Yes	
5A1		141.1		92.53	138.9	91.08							
5A2		141.2		92.59									
5B1		141.6		92.86	138.6	90.89							
5B2		140.8		92.33									
5C1	4/3/2009	138.8		91.02	138.4	90.76							
5C2		139.5		91.48									
5D1		139.2		91.28	142	93.12							
5D2		140.2		91.94									
5E1		138.5		90.82	137.2	89.97							
5E2		139.2	2.450	91.28			10	5	1.33	3.01	Pass	Yes	

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F & t Gmm Spreadsheet

HMA 1R 602 07-06006-R1

Lots:	1	10	**Project #**	KA 1441-01		**Name of QC Tester**		
Dates:	3/25/2009	4/29/2009	**Contract #**	508106343		**Certification # of QC Tester**		
Mix Type:		SR-12.5A	64-28	HMA Overlay				

Lot	Date	Gmm Contractor Quality Control Tests (Gmm)	KDOT Verification Test (Gmm)	Number of Contractor Tests	Number of KDOT Tests	t Test	t(crit)	Are Means The Same?	Use Contractor Test Results?	Comments
1A	3/25/2009	2.428		1	0			Pass	Yes	
1B	3/31/2009	2.447		2	0					
1C	3/31/2009	2.448		3	0			Pass	Yes	
1D	4/02/2009	2.436	2.435	4	1					
1E										
1F										
2A	4/02/2009	2.447		5	1			Pass	Yes	
2B	4/03/2009	2.448		6	1					
2C	4/03/2009	2.451	2.453	7	2					
2D	4/03/2009	2.450		8	2			Pass	Yes	
2E										
2F										
3A	4/07/2009	2.455		9	2					
3B	4/07/2009	2.454		10	2			Pass	Yes	
3C	4/08/2009	2.445		11	2					
3D	4/08/2009	2.451	2.453	12	3	0.06	3.01			
3E										
3F										
4A	4/08/2009	2.450		13	3	0.02	2.98	Pass	Yes	
4B	4/09/2009	2.446	2.450	14	4	0.21	2.92	Pass	Yes	
4C	4/10/2009	2.444		15	4	0.27	2.90			
4D	4/10/2009	2.437		16	4	0.41	2.88	Pass	Yes	
4E										
4F										
5A	4/13/2009	2.441		17	4	0.49	2.86			
5B	4/13/2009	2.452	2.440	18	5	0.02	2.83	Pass	Yes	
5C	4/14/2009	2.443		19	5	0.07	2.82			
5D	4/14/2009	2.449		20	5	0.03	2.81	Pass	Yes	
5E										
5F										

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F & t Compressive Strength Spreadsheet

Lots:			Project #	K 7892-01	Name of QC Tester		Junior Samples
Dates:	5/16/08		Contract #	509111888	Certification # of QC Tester		BR-549

			Compressive Strength Comparison						
Lot	Date	Corrected Contractor Compressive Strength (MPa)	Corrected KDOT Compressive Strength (MPa)	Number of Contractor Tests	Number of KDOT Tests	T Test	T(crit)	Are Means The Same?	Use Contractor Test Results?
1A1	5/16/2008	48.82	42.89						
1A2									
1B1		40.54							
1B2									
1C1		38.31							
1C2									
1D1		48.46							
1D2									
1E1		48.64							
1E2				5	1			Pass	Yes
2A1	5/17/2008	38.20	38.58						
2A2									
2B1		38.94							
2B2									
2C1		41.73							
2C2									
2D1		39.93							
2D2									
2E1									
2E2				4	1			Pass	Yes
3A1	7/22/2008	41.58	31.40						
3A2									
3B1		35.70							
3B2									
3C1		34.04							
3C2									
3D1		40.47							
3D2									
3E1									
3E2				13	3	1.13	2.98	Pass	Yes
4A1	7/23/2008	43.90	37.97						
4A2									
4B1		36.81							
4B2									
4C1		45.97							
4C2									
4D1		41.13							
4D2									
4E1		36.19							
4E2				18	4	1.34	2.85	Pass	Yes
5A1	7/24/2008	41.72	40.02						
5A2									
5B1		42.09							
5B2									
5C1		40.41							
5C2									
5D1		43.04							
5D2									
5E1									
5E2				22	5	1.49	2.79	Pass	Yes

Revised 2016

F & t Thickness Spreadsheet

SS2007 Section 501								
Lots:			**Project #**	K 7892-01	**Name of QC Tester**		Junior Samples	
Dates:	5/16/08		**Contract #**	509111888	**Certification # of QC Tester**		BR-549	

Concrete Thickness Comparison									
Lot	Date	Contractor Core Length (mm)	KDOT Core Length (mm)	Number of Contractor Tests	Number of KDOT Tests	T Test	T(crit)	Are Means The Same?	Use Contractor Test Results?
1A1	5/16/2008	265	257						
1A2									
1B1		261							
1B2									
1C1		261							
1C2									
1D1		241							
1D2									
1E1		256							
1E2				5	1			Pass	Yes
2A1	5/17/2008	272	266						
2A2									
2B1		264							
2B2									
2C1		264							
2C2									
2D1		275							
2D2									
2E1									
2E2				4	1			Pass	Yes
3A1	7/22/2008	280	270						
3A2									
3B1		266							
3B2									
3C1		260							
3C2									
3D1		238							
3D2									
3E1									
3E2				13	3	0.36	2.98	Pass	Yes
4A1	7/23/2008	275	269						
4A2									
4B1		277							
4B2									
4C1		268							
4C2									
4D1		260							
4D2									
4E1		264							
4E2				18	4	0.31	2.85	Pass	Yes
5A1	7/24/2008	280	278						
5A2									
5B1		280							
5B2									
5C1		267							
5C2									
5D1		278							
5D2									
5E1									
5E2				22	5	0.37	2.79	Pass	Yes